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Original article

A note on computation of pseudospectra

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Abstract

The aim is to contribute to pseudospectra computation via a path following technique. Given a matrix $A \in \mathbb{C}^{n\times n}$, we compute the branch consisting of a fixed singular value ϵ and corresponding left and right singular vectors of the parameter dependent matrix $(x+\mathrm{i}y)I-A$. The fact that the branch corresponds to the smallest singular value $\sigma_{\min}((x+\mathrm{i}y)I-A)=\epsilon$ is sufficient to verify at just one point of the branch due to the continuity argument. We can exploit a standard ready-made software.

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1. Introduction

In applications concerning linear processes represented by a non-normal matrix $A \in \mathbb{C}^{n\times n}$, the classical spectral information may be misleading. Instead, the information concerning ϵ-*pseudospectra* yields a deeper insight. For the motivation, see e.g. the monograph [16].

Out of equivalent definitions of the pseudospectra we stick to the following one: Let $A \in \mathbb{C}^{n\times n}$. Given $\epsilon > 0$,

$$\Lambda_\epsilon \equiv \{z \in \mathbb{C} \,:\, \sigma_{\min}(zI - A) < \epsilon\} \;, \tag{1}$$

where $\sigma_{\min}(zI-A)$ denotes the smallest singular value of $zI-A$ and I is the identity matrix.

We discuss computing pseudospectra. As the basic computational tool, variants of grid techniques are used:

1. Construct a mesh D in the complex plane $\mathbb{C}$ which envelopes the required part of Λ_ϵ for selected values of ϵ.
2. Compute $\sigma_{\min}(zI-A)$ at each grid point $z \in D$.
3. Consider the level sets (2) for the selected values of ϵ. Visualize them as the contour plots on the grid:

$$\partial\Lambda_\epsilon = \{z \in \mathbb{C} \,:\, \sigma_{\min}(zI - A) = \epsilon\} \;. \tag{2}$$

As step 2 is concerned, iterative techniques are recommended: *inverse iterations* to compute the smallest eigenvalue of $(zI-A)^*(zI-A)$ see e.g. [11], or *inverse Lanczos iterations* which approximate the minimal singular value of

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