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Mathematics and Computers in Simulation 82 (2012) 1069–1078

MATHEMATICS AND COMPUTERS IN SIMULATION

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Hidden solitons in the Zabusky–Kruskal experiment: Analysis using the periodic, inverse scattering transform

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Received 18 October 2009; received in revised form 9 May 2010; accepted 17 May 2010
Available online 4 June 2010

Abstract

Recent numerical work on the Zabusky–Kruskal experiment has revealed, amongst other things, the existence of hidden solitons in the wave profile. Here, using Osborne's nonlinear Fourier analysis, which is based on the periodic, inverse scattering transform, the hidden soliton hypothesis is corroborated, and the *exact* number of solitons, their amplitudes and their reference level is computed. Other "less nonlinear" oscillation modes, which are *not* solitons, are also found to have nontrivial energy contributions over certain ranges of the dispersion parameter. In addition, the reference level is found to be a non-monotone function of the dispersion parameter. Finally, in the case of large dispersion, we show that the one-term nonlinear Fourier series yields a very accurate approximate solution in terms of Jacobian elliptic functions.

PACS: 05.45.Yv; 02.30.Ik

MSC: 35Q51; 35P25

Keywords: Hidden solitons; Korteweg–de Vries equation; Inverse scattering transform; Nonlinear Fourier analysis

1. Introduction

Four decades after the discovery of solitons by Zabusky and Kruskal (ZK) [35] through a computational experiment, the study of the evolution of harmonic initial data under the Korteweg–de Vries (KdV) equation on a periodic interval is far from complete [4]. Beyond the discovery [35] of the ability of the KdV equation's (localized) traveling-wave solutions (termed 'solitons') to retain their "identity" (shape, speed) after collisions, modern numerical simulations by Salupere et al. of this paradigm equation of solitonics reveal such exotic features as "hidden" (or "virtual") solitons [9], emergence of soliton ensembles [32] and long-time periodic patterns of the trajectories [29,33]. These phenomena have been shown to be generic of nonlinear waves, as they occur under other governing equations as well [13,27,28].

This raises the simple, yet quite fundamental, question: How many solitons emerge from a harmonic input? A successful approach to answering this question is based upon *discrete spectral analysis* [10,30,31]. The essence of this method is to characterize the solitary waves based on the information inherent in the pseudospectral numerical approximation of the underlying partial differential equation (PDE) [26]. In general, solitary wave identification is

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doi:10.1016/j.matcom.2010.05.021