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# A Boussinesq model for pressure and flow velocity waves in arterial segments

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**Abstract**

We derive a nonlinear model for the pressure and flow velocity wave propagation in an arterial segment. We then study the transmission and reflection of pulses at bifurcation. We observe a linear dependence of the transmitted speeds to the incoming speeds, and similarly for the reflected speeds. We propose a method for validating the numerical results obtained from this model against real data.


*Keywords:* Boussinesq-type system; Reflection and transmission coefficients; Pressure waves in arteries

## 1. Introduction

This paper describes a mathematical model for pulse and flow velocity wave propagation in an arterial segment and at bifurcations. This study is motivated by the previous work [1] in the context of applications to latency measurements, such as those described in [6]. In particular, it is geared towards modeling the complex relationship between the cerebral flow velocity (CBFV) and arterial blood pressure (ABP) latency measurements during dynamic states in the cardiovascular system.

Using a common simplifying assumption, we model arteries as a fluid-filled compliant tube with elastic walls. At each spatial location $x$ in the vessel and each time $t \geq 0$ we observe the transmural pressure $p = p(x, t)$ (constant across the arterial vessel) and flow velocity $u = u(x, t)$. This flow velocity is usually taken to be the cross-sectional average of the axial velocities in the fluid, while the flow is assumed to be axisymmetric. Radial velocities are typically obtained from the incompressibility assumption on the fluid. Finally, under the axisymmetric assumption, we denote $\eta = \eta(x, t)$ the wall displacement and $A = A(x, t)$ the cross-sectional area at location $x$ and time $t$. The flow rate across a fixed Cross-section of the tube is then $Q = Au$. Also, note that $A = \pi(\eta + r_0)^2$ (Fig. 1).

Conservation of mass and momentum leads to the system of 1D-equations for $A$, $u$ (see e.g. [4]):

$$A_t + (Au)_x = 0 \tag{1}$$

$$u_t + uu_x + \frac{1}{\rho} p_x = f \tag{2}$$

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