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Original article

Analytical solution of gaseous slip flow between two parallel plates described by the Oseen equation

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Abstract

This paper is focused on the derivation of analytical solution describing the development of gas pressure driven microflow in a gap between two parallel plates. The gas flow is assumed to be steady, laminar and incompressible. For the mathematical description of the problem, the Oseen flow model is used. The first-order velocity slip boundary conditions are considered at the walls of the gap. The analytical solution for the velocity profile development is obtained using the method of Laplace transformation. The applicability of the Oseen flow model is analysed on two test cases in this study: pressure driven microflow of argon and pressure driven airflow with $\mathrm{Kn} \to 0$ for which the analytical solution is compared with the numerical one.

Keywords: Pressure driven microflow; Slip flow regime; Oseen equation; Velocity profile development; Analytical solution

1. Introduction

Flow and heat transfer in very narrow gaps and microchannels are the freguently mentioned topics of theoretical studies in recent years. Problems of microflow occur in various technical systems and devices such as heat exchangers, nuclear reactors and microturbines as well as in a number of biological systems (capillaries, brain, lungs, kidneys, etc.). In such problems, the flow is assumed to be laminar because the characteristic dimension (in many cases the hydraulic diameter D_h) is very small resulting in Reynolds number Re much lower than its critical value determining the transition from laminar to turbulent flow. When dealing with microfluidics, influence of the Knudsen number Kn has to be taken into account very often. This important dimensionless parameter can be defined as $\mathrm{Kn} = \lambda/D_h = \mathrm{Ma}/\mathrm{Re}\sqrt{(\pi\gamma/2)}$ [8,9], where λ is the molecular mean free path, Ma is the Mach number and γ is the specific heat ratio.

Depending on the value of the Knudsen number Kn, the character of microflows can be divided into four flow regimes [8,9]: the continuum flow regime for $\mathrm{Kn} < 10^{-3}$, the slip flow regime for $10^{-3} < \mathrm{Kn} < 10^{-1}$, the transition flow regime for $10^{-1} < \mathrm{Kn} < 10$ and the free molecular flow regime for $\mathrm{Kn} > 10$. Particularly, the slip flow regime occurs in a number of microscale applications and is the object of interest also in our study. For the mathematical modelling of

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