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Stability analysis for a fractional differential model of HIV infection of CD4$^+$ T-cells with time delay

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Abstract

In this paper, we introduce fractional-order derivatives into a model of HIV infection of CD4$^+$ T-cells with time delay. We deal with the stability of both the viral free equilibrium and the infected equilibrium. Criteria are given to ensure that both the equilibria are asymptotically stable for all delay under some conditions. Numerical simulations are carried out to illustrate the results.

Keywords: HIV infection; Fractional-order; Time delay; Asymptotic stability

1. Introduction

Fractional calculus is a generalization of classical differentiation and integration to arbitrary (non-integer) order. In recent years fractional calculus has been a fruitful field of research in science and engineering [12,23,25,26,30]. In fact, many researchers are currently paying attention to the fractional calculus concepts and we can refer its adoption in chemistry, electromagnetic waves [8], quantitative finance [16], quantum evolution of complex system [21], chaos and fractals [7], robotics [31], control systems [20], etc. Thus, as mentioned in [23], there is no field that has remained untouched by fractional derivatives.

In particular, fractional differential equations as an important research branch of fractional calculus gain much attention. Many results on local existence, uniqueness and structural stability of solutions of specific fractional differential equations are successively established [10,17,32,37]. Also varieties of schemes for numerical solutions of fractional differential equations are proposed [15,19]. Meanwhile, the applications of fractional differential equations to physics, biology and engineering are a recent focus of interest [9,12]. Many systems are known to display fractional-order dynamics, such as viscoelastic systems [4,13], electrode–electrolyte polarization [11] and complex adaptive systems in biology [2].

Human immunodeficiency virus (HIV) is a lentivirus (a member of the retrovirus family) that causes acquired immunodeficiency syndrome (AIDS) [34], a condition in humans in which the immune system begins to fail, leading to life-threatening opportunistic infections. HIV infects primarily vital cells in the human immune system such as helper T-cells (to be specific, CD4$^+$ T-cells), macrophages, and dendritic cells. When CD4$^+$ T-cell numbers decline below a

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