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# A shape optimization method for nonlinear axisymmetric magnetostatics using a coupling of finite and boundary elements

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**Abstract**

In this paper we propose a method for constrained shape optimization governed with a nonlinear axisymmetric magnetostatic state problem and we apply it to an optimal shape design of an electromagnet. The state problem is solved via Hiptmair's symmetric coupling of finite elements employed in the interior ferromagnetic domain and boundary elements modelling the exterior air domain as well as current excitations. As a novelty we derive Duffy regularization transforms of the boundary element integrals for the axisymmetric case, which are then evaluated using a tensor-product Gaussian quadrature. Nonlinear ferromagnetic behaviour is resolved by Newton iterations. The optimization method under both linear and nonlinear constraints relies on the active-set steepest-descent search, projections onto the set of linearized constraints, and an adjoint method of shape sensitivity analysis. Shape perturbations influence grid deformation via a solution to an auxiliary torsion-free linear elasticity problem. Finally, numerical results are presented.


*Keywords:* Shape optimization; Nonlinear magnetostatics; Boundary element method; Finite element method; Adjoint sensitivity analysis

## 1. Introduction

Shape optimization has become a standard computing tool in both research and engineering design. It concerns on minimization of a given quantity, called cost functional, under some restrictions, called constraints, so that their evaluation involves a computation of a physical field typically described by partial differential equations, called state problem, over geometrical domains the shape or interfaces of which are variable. There is a couple of classification of shape optimization methods. One class of methods [9] approximates the original inifinite-dimensional optimization problem by a sequence of its finite-dimensional counterparts. Then, necessary optimality (Karush–Kuhn–Tucker) conditions for the subproblems are formulated separately in terms of linear algebra and gradient-type solution methods are employed. Another approach [23] aims to develop first an infinite-dimensional setting of the necessary optimality conditions in terms of shape derivatives and then discretize them properly. For some problems one can even derive sufficient optimality conditions in terms of a shape Hessian in the infinite-dimension and prove its coercivity [6], in order to employ Newton-type optimization methods. A nice property of the latter methods is that the optimality

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