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A fast simulation method for 1D heat conduction[☆]

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Abstract

A flexible solution method for the initial-boundary value problem of the temperature field in a one-dimensional domain of a solid with significantly nonlinear material parameters and radiation boundary conditions is proposed. A transformation of the temperature values allows the isolation of the nonlinear material characteristics into a single coefficient of the heat conduction equation. The Galerkin method is utilized for spatial discretization of the problem and integration of the time domain is done by constraining the boundary heat fluxes to piecewise linear, discontinuous signals. The radiative heat exchange is computed with the help of the Stefan–Boltzmann law, such that the ambient temperatures serve as system inputs. The feasibility and accuracy of the proposed method are demonstrated by means of an example of heat treatment of a steel slab, where numerical results are compared to the finite difference method.

Keywords: Heat conduction; Nonlinear material parameters; Method of weighted residuals; Galerkin method; Radiative heat exchange

1. Introduction

In process control applications, there is a need for mathematical models which are both computationally inexpensive as well as reliable in terms of accuracy and convergence. These requirements are particularly important for models to be used in real-time applications like trajectory planning, optimization, or control. Motivated by these needs, a method to determine the transient temperature field in a one-dimensional domain of a solid with significantly nonlinear material parameters and radiation boundary conditions is proposed. The approach originates from an application in the steel industry, where slabs or rolled products are to be heat-treated or reheated according to specific temperature trajectories [2,21,22]. However, by analogy, the method can be applied to other diffusion-convection systems described by parabolic initial-boundary value problems.

The paper is organized as follows: Section 2 starts with a brief review of the heat conduction equation (strong formulation) with nonlinear material parameters and Neumann boundary conditions, followed by a transformation of the temperature such that the nonlinearity is isolated into a single parameter of the parabolic problem. Thereupon, the problem is restated in the weak formulation, which is suitable for the method of weighted residuals (MWR). Here, the Galerkin method (GM) is employed to derive a low-dimensional lumped-parameter system, and a time integration

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