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Original Article

High order well-balanced scheme for river flow modeling☆

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Abstract

We propose a new numerical scheme based on the finite volumes to simulate the river flow in the presence of a variable bottom surface. Our approach is based on the Riemann solver designed for the augmented quasilinear homogeneous formulation. The scheme has general semidiscrete wave-propagation form and can be extended to an arbitrary high order accuracy. The main goal is to construct the scheme, which is well-balanced, i.e. maintains not only some special steady states, but all steady states which can occur.

Keywords: Hyperbolic conservation laws; Finite volume methods; steady states; semidiscrete schemes

1. Introduction

We are interested in solving the problem related to the fluid flow through the channel with the general cross-section area described by

$$a_t + q_x = 0, \quad q_t + \left(\frac{q^2}{a} + gI_1\right)_x = -gab_x + gI_2. \tag{1}$$

Here $a = a(x, t)$ is the unknown cross-section area, $q = q(x, t)$ is the unknown discharge, $b = b(x)$ is the function of elevation of the bottom, g is the gravitational constant and

$$I_1 = \int_0^{h(x,t)} [h(x, t) - \eta]\sigma(x, \eta)d\eta, \tag{2}$$

$$I_2 = \int_0^{h(x,t)} (h(x, t) - \eta)\left[\frac{\partial\sigma(x, \eta)}{\partial x}\right] d\eta, \tag{3}$$

where η is the depth integration variable, $h(x, t)$ is the water depth and $\sigma(x, \eta)$ is the width of the cross-section at the depth η. The derivation can be found in [4].

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