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The generation of capillary-gravity solitary waves by a surface pressure forcing

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Abstract

A weakly nonlinear model is used to study capillary-gravity waves generated by a traveling localized surface pressure distribution. The weakly nonlinear model is a truncation of the potential flow equations in deep water, and includes cubic nonlinear terms. Numerically, solitary waves are shown to be generated by a near-monochromatic, subcritical forcing. The presence of these solitary waves is predicted using a forced nonlinear Schrödinger equation.

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Keywords: Solitary wave; Surface tension; Nonlinear Schrödinger

1. Introduction

In this paper, the generation of capillary-gravity waves by a moving pressure distribution is investigated. We observe that solitary waves can be generated by a subcritical forcing. A family of weakly nonlinear model equations for capillary-gravity water waves is derived from the potential flow equations. Model equations from this family can be characterized by the highest degree in their nonlinear term. We study the forced dynamics of a cubic member of this family.

Understanding forced water waves has been of interest for over a century. Steady flow past an obstacle in the potential flow equations was studied famously by Rayleigh [38] – more recent examples include [33,42]. The dynamic, forced problem has been studied in a number of weakly nonlinear models which include long wave (or shallow water) assumptions, for example the KdV, Benjamin, Benjamin-Ono, and BBM equations [9,6,12,23,24,29,31]. Unforced capillary-gravity wave dynamics have been studied using weakly nonlinear models which do not include a long wave assumption – an overly restrictive assumption for waves at finite wavenumber [2–4,8]. In this paper, the *isotropic* deep water capillary-gravity wave model of [3] is generalized to include cubic terms and a surface pressure distribution. The cubic model Eq. (2.5) is used to study the dynamic problem of flow in the wake of a surface pressure distribution, a simple model for the generation of water waves by wind.

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