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A parareal algorithm based on waveform relaxation☆

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Abstract

We report a new parallel iterative algorithm for time-dependent differential equations by combining the known waveform relaxation (WR) technique with the classical parareal algorithm. The parallelism can be simultaneously exploited in both sub-systems by WR and time by parareal. We also provide a sharp estimation on errors for the new algorithm. The iterations of parareal and WR are balanced to optimize the performance of the algorithm. Furthermore, the parallel speedup and efficiency of the new approach are analyzed by comparing with the classical parareal algorithm and the WR technique, respectively. Numerical experiments are carried out to verify the effectiveness of the theoretic work.

MSC: 45L05; 65L05; 65L20

Keywords: Parareal algorithm; Waveform relaxation; Parallelism in sub-systems and time; Convergence analysis

1. Introduction

The parareal algorithm is a time-parallel iterative process for solving differential equations, which was first proposed by Lions et al. [18]. It has been applied to many practical problems [3,5,6,20–22,27]. Several efficient variants in such parallel framework have been presented [9,12,13,23]. The parareal algorithm consists of some individual sub-problems with their own initial values which are updated during iterations. Fortunately, after few iterations its global accuracy of computations is often comparable to that given by a sequential numerical method used on a fine discretization in time.

Waveform relaxation (WR), also known as dynamic iteration, was first presented as a practical computational tool by Lelarasmee et al. to simulate MOS VLSI circuits [17]. It is also a highly parallel iterative technique for numerically solving large scale time-dependent systems, such as ordinary differential equations (ODEs) [4,8,10] and differential algebraic equations (DAEs) [14–16]. The WR technique consists of a "divide-and-conquer" approach, which means it decouples a large system into a number of simplified sub-systems on time-interval, and these sub-systems can be solved independently with their own time integrations.

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