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Wave equations with super-critical interior and boundary nonlinearities

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Abstract

This article presents a unified overview of the latest, to date, results on boundary value problems for wave equations with *super-critical* nonlinear sources on *both* the interior and the boundary of a bounded domain $\Omega \in \mathbb{R}^n$. The presented theorems include Hadamard local wellposedness, global existence, blow-up and non-existence theorems, as well as estimates on the uniform energy dissipation rates for the appropriate classes of solutions.

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1. Introduction

1.1. The model and the problem

A rich body of research literature to date addresses semi-linear boundary value problems for the wave equation:

$$\begin{cases} u_{tt} - \Delta u + g_0(u_t) = f(u) \quad \text{in } \Omega \times [0, \infty), \\ \partial_\nu u + u + g(u_t) = h(u) \quad \text{on } \Gamma \times [0, \infty), \\ \{u(0), u_t(0)\} = \{u_0, u_1\} \in H^1(\Omega) \times L_2(\Omega), \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded open set with sufficiently smooth boundary Γ. Monotone continuous feedback maps $g_0(s)$, $g(s)$ model respectively the interior and boundary damping terms. The Nemytski operators associated with differentiable scalar maps f and h represent source terms, and do not necessarily satisfy any a priori dissipativity conditions.

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