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Wave front behavior of traveling wave solutions for a PDE having square-root dynamics

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Abstract

Our main goal is to investigate the asymptotic behavior of traveling wave solutions to a nonlinear parabolic PDE having square-root dynamics in its reaction term. To calculate this result, we apply the method of dominant balance.

Keywords: Parabolic PDE's; Traveling waves; Asympotitics

1. Introduction

Nonlinear parabolic partial differential equations [1,4,5,9] provide models for important physical and engineering systems. Particular examples include disease transmission [10,12], plasma physics [15], diffusion [4], and porous media [6]. One of the most studied such equations is the Fisher equation [5,9,12]

$$u_t = u_{xx} + u - u^2, \tag{1.1}$$

$$u = u(x, t), \quad t \geq 0, \quad -\infty < x < \infty, \tag{1.2}$$

where the equation is written in dimensionless form. Note that this PDE has two fixed-points (constant solutions)

$$\bar{u}^{(1)} = 0, \quad \bar{u}^{(2)} = 1, \tag{1.3}$$

and they are, respectively, unstable and stable. Further, the Fisher equation has a traveling wave solution [5,9,12] of the form

$$u(x, t) = f(x - ct) = f(z), \quad z = x - ct, \tag{1.4}$$

where c is the velocity, satisfying the restriction [9]

$$c \geq 2. \tag{1.5}$$

The function $f(z)$ has the following properties:

$$\text{Lim}_{z\to-\infty} f(z) = 1, \qquad \text{Lim}_{z\to+\infty} f(z) = 0, \tag{1.6a}$$

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