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On the existence of high Lewis number combustion fronts

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Abstract

We study a mathematical model for high Lewis number combustion processes with the reaction rate of the form of an Arrhenius law with or without an ignition cut-off. An efficient method for the proof of the existence and uniqueness of combustion fronts is provided by geometric singular perturbation theory. The fronts supported by the model with very large Lewis numbers are small perturbations of the front supported by the model with infinite Lewis number.
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Keywords: Geometric singular perturbation theory; Combustion fronts; Ignition cut-off

1. The model

We consider a well-known model for the propagation of combustion waves in the case of premixed fuel, with no heat loss, in one spatial dimension $x \in \mathbb{R}$. The system describing evolution of the temperature u and concentration of the fuel y reads

$$\begin{aligned} u_t &= u_{xx} + y\Omega(u), \\ y_t &= \varepsilon y_{xx} - \beta y \Omega(u). \end{aligned} \tag{1}$$

We will first describe the parameters of the system and then discuss the reaction terms. The system has two parameters. One is the exothermicity parameter $\beta > 0$ which is the ratio of the activation energy to the heat of the reaction. The other is the reciprocal of the Lewis number $\varepsilon = 1/\mathrm{Le} > 0$. Therefore, ε represents the ratio of the fuel diffusivity to the heat diffusivity. The system has been studied for various parameter regimes. Of interest to us are traveling wave solutions to (1) in two cases. One is the system (1) with $\varepsilon = 0$ ($\mathrm{Le} = \infty$). Its physical prototype is the combustion of solid fuels, more precisely, combustion that involves the solid phase only with no gaseous products present. The other is the case of $0 < \varepsilon \ll 1$, i.e., when Le is very large but finite. This situation is also physical: (1) then describes burning of very high density fluids at high temperatures.

The interest in the relation between cases with zero and nonzero ε is explained by two facts. First is that during the burning of solid fuels some liquefaction of the fuel might occur in the reaction zone, thus causing a nonzero value of

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