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On n-fold integral filters in triangle algebras

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Abstract. In this paper, we introduce the notion of n-fold IVRL-extended integral filter in triangle algebras and give some properties of it. The relationship between this IVRL-filters and some types of IVRL-filters are determined. Also, we prove that every proper IVRL-filter F which satisfies in n-fold double negation. If F is n-fold IVRL-extended integral filter and n-fold IVRL-extended fantastic filter, then F is n-fold IVRL- extended obstinate filter.

Keywords: Triangle algebra, IVRL-filter, integral filter.

1. Introduction and Preliminaries

The concept of a residuated lattice was firstly introduced by M. Ward and R. P. Dilworth as generalization of ideal lattices of rings. The properties of a residuated lattice were presented in [1] and the lattice of filters of a residuated lattice was investigated in [2]. Van Gass et al. introduced triangle algebras as a variety of residuated lattices equipped with approximation operators and with a third angular point u, different from 0,1 [4]. They defined some types of filters in triangle algebras and obtained some interesting results [3].

In the present paper, we are going to study triangle algebras by means of n-fold IVRL-extended integral filter. For this purpose, we define n-fold integral IVRL-filters and prove some of their poperties. Next, the basic properties of n-fold integral IVRL-filters are considered.

Definition 1.1. *[4] A residuated lattice is an algebra $\mathcal{L} = (L, \vee, \wedge, *, \to, 0, 1)$ with four binary operations and two constants 0,1 such that:*

- *$(L, \vee, \wedge, 0, 1)$ is a bounded lattice,*
- *$*$ is commutative and associative, with 1 as neutral element, and*
- *$x * y \leq z$ if and only if $x \leq y \to z$, for all x, y and z in L.*

Definition 1.2. *[4] Given a lattice $\mathcal{A} = (A, \vee, \wedge)$, its triangularization $\mathbb{T}(\mathcal{A})$ is the structure $\mathbb{T}(\mathcal{A}) = (Int(\mathcal{A}), \vee, \wedge)$ defined by*

- *$Int(\mathcal{A}) = \{[x_1, x_2] : (x_1, x_2) \in A^2$ and $x_1 \leq x_2\}$,*
- *$[x_1, x_2] \wedge [y_1, y_2] = [x_1 \wedge y_1, x_2 \wedge y_2]$,*
- *$[x_1, x_2] \vee [y_1, y_2] = [x_1 \vee y_1, x_2 \vee y_2]$.*

The set $D_{\mathcal{A}} = \{[x, x] : x \in L\}$ is called the diagonal of $\mathbb{T}(\mathcal{A})$.

Definition 1.3. *[4] An interval-valued residuated lattice (IVRL) is a residuated lattice $(Int(\mathcal{A}), \vee, \wedge, \odot, \to_{\odot}, [0, 0], [1, 1])$ on the triangularization $\mathbb{T}(\mathcal{A})$ of a bounded lattice $\mathcal{A}$, in which the diagonal $D_{\mathcal{A}}$ is closed under $\odot$ and $\to_{\odot}$, i.e. $[x, x] \odot [y, y] \in D_{\mathcal{A}}$ and $[x, x] \to_{\odot} [y, y] \in D_{\mathcal{A}}$, for all x, y in L.*

Definition 1.4. *[4] A triangle algebra is a structure $\mathcal{A} = (A, \vee, \wedge, *, \to, \nu, \mu, 0, u, 1)$ in which $(A, \vee, \wedge, *, \to, 0, 1)$ is a residuated lattice, ν and μ are unary operations on A, u a constant, and satisfying the following conditions:*

$(T.1)\nu x \leq x,$	$(T.1^{'})x \leq \mu x,$
$(T.2)\nu x \leq \nu\nu x,$	$(T.2^{'})\mu\mu x \leq \mu x,$
$(T.3)\nu(x \wedge y) = \nu x \wedge \nu y,$	$(T.3^{'})\mu(x \wedge y) \leq \mu x \wedge \mu y,$
$(T.4)\nu(x \vee y) = \nu x \vee \nu y,$	$(T.4^{'})\mu(x \vee y) \leq \mu x \vee \mu y,$
$(T.5)\nu u = 0,$	$(T.5^{'})\mu u = 1,$
$(T.6)\nu\mu x = \mu x,$	$(T.6^{'})\mu\nu x = x,$