Fuzzy primary filters in BL-algebras

FereshtehForouzesh*
*Faculty of mathematics and computing Higher Education Complex of Bam

Abstract: *In this paper, we introduce the notion of fuzzy primary filter in BL-algebras. Some properties of fuzzy primary filters are given. Also, characterization of fuzzy primary filters is given. Also we prove that primary extension property for fuzzy filters.*

Keywords: BL-algebra, fuzzy filter, fuzzy primary filter.

1. Introduction

BL-algebras have been introduced by Hájek [2] in order to investigate many valued logic by algebraic means. He provide an algebraic proof of the completeness theorem of ‘‘Basic Logic’’ (BL, for short) arising from the continuous triangular norms, familiar in the fuzzy logic framework.

Filters theory play an important role in studding these algebras. From logical point of view, various filters correspond to various sets of provable formula. Hájek [2] introduced the notions of filters and prime filters in BL-algebras. Using prime filters of BL-algebras, he proved the completeness of basic logic BL. Turunnen [7-9] studied some properties of the filters and prime filters of BL-algebras. In [6], he introduced the notion of Boolean filters in BL-algebras and derived some characterizations of Boolean filters. He also proved that a BL-algebra is bipartite if and only if it has a proper Boolean filter.

In [3], Haveshki et al. continued an algebraic analysis of BL-algebras and they introduced positive implicative filters of BL-algebras.

The concept of fuzzy sets was introduced by Zadeh [10]. Liu et al. introduced fuzzy filters of BL-algebras [4]. Then they introduced fuzzy Boolean and positive implicative filters of BL-algebras [5].

In this paper, we introduce the notion of fuzzy primary filter in BL-algebras. Some properties of fuzzy primary filters are given. Also, characterization of fuzzy primary filters is given. Also we prove that primary extension property for fuzzy filters.

In the sequel, A is a BL-algebra.

We recall that some definitions and results which will be used in the following:

2. Preliminaries

Definition 1.1 [2] A BL-algebra is a structure $(A,\wedge,\vee,\odot,\rightarrow,0,1)$ such that

(i) $(A,\wedge,\vee,0,1)$ is a bounded lattice,

(ii) $(A,\odot,1)$ is an abelianmonoid, i.e. $\odot$ is commutative and associative

(iii) $x\odot 1=1\odot x=x$,

The following conditions hold for all $x,y,z\in A$:

(B1) $x\odot y\leq z$ if and only if $x\leq y\rightarrow z$,

(B2) $x\wedge y=x\odot(x\rightarrow y)$,

(B3) $(x\rightarrow y)\vee(y\rightarrow x)=1$.

Lemma 1.2[2] Let A be a BL-algebra. The following properties hold:

(1) $x\leq y$ if and only if $x\rightarrow y=1$,

(2) $x\rightarrow(y\rightarrow z)=(x\odot y)\rightarrow z$,

(3) $x\odot y\leq x\wedge y$,

(4) $(x\rightarrow y)\odot(y\rightarrow z)\leq x\rightarrow z$,

(5) $x\vee y=((x\rightarrow y)\rightarrow y)\wedge((y\rightarrow x)\rightarrow x)$,

(6) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$,

(7) $x\rightarrow y\leq(z\rightarrow x)\rightarrow(z\rightarrow y)$,

(8) $x\odot x^*=0$ and $x^*\leq x\rightarrow y$,

(9) $1\rightarrow x=x$, $x\rightarrow x=1$, $x\leq y\rightarrow x$, $x\rightarrow 1=1$.

Definition1.3 [2] Let X and Y are BL-algebras. A function $f:X\rightarrow Y$ is called homomorphism of BL-algebras if and only if

(1) f(0)=0, f(1)=1,

(2) $f(x\odot y)=f(x)\odot f(y)$,