# Estimation with Heterogeneous Sequential *k*-out-of-*n* System Lifetimes

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***Abstract:*** *When the component lifetimes are independent and identically distribution, the order statistics are suitable for describing the system lifetime. Notice that here failing a component does not change the distribution functions of lifetimes of surviving components. Sequential order statistics, is an extension of the usual order statistics and used for modeling lifetimes of sequential r -out-of- n systems, coming from heterogeneous exponential distributions are considered. The problem of hypotheses testing for exponential populations on the basis of multiple independent sequential order statistics samples under a conditionally proportional hazard rate model via a Bayesian and classical approach is studied. Via the likelihood approach, statistical procedures including estimation, either point or interval of the parameters as well as the generalized likelihood ratio are derived for testing homogeneity of the exponential populations. Approximate confidence intervals and Fisher Information are derived on the basis of observed multiply system lifetimes.*

**Keywords:** Estimation; hypotheses testing; $r$ -out-of- $n$ systems; Order Statistics; Bayesian approach

## 1. Introduction

Let $X_1,\cdots,X_n$ be independent and identically distributed (i.i.d.) random variables with a common distribution function (DF), say $F$, and abbreviated by $X_1,\cdots,X_n \overset{i.i.d.}{\sim} F$. Denote in magnitude order of $X_1,\cdots,X_n$ by $X_{1:n} \le \cdots \le X_{n:n}$, known as order statistics. The theory of order statistics has been used widely in literature. For example, in system reliability analyses, lifetimes of $r$ -out-of- $n$ systems coincide to $X_{r:n}$ where $X_1,\cdots,X_n$ stand for component lifetimes. For more information, See Barlow *et al.*[1] and David *et al.*[7] and references therein. When the component lifetimes are $i.i.d.$, the order statistics are suitable for describing the system lifetime. Notice that here failing a component does not change the DFs of lifetimes of surviving components. Motivated by Cramer *et al.* [8] and Cramer *et al.* [9] the failure of a component may result in a higher load on the surviving components and hence causes the lifetime distributions change. They introduced the concept of sequential order statistics which may be adequate for modelling these systems. More precisely, suppose that $F_j$, for $j=1,\cdots n$, denotes the common DF of the component lifetimes when $n-j+1$ components are working. The components begin to work independently at time $t=0$ with the common DF $F_1$. When at time $x_1$, the first component failure occurs, the remaining $n-1$ components are working with the (truncate) common DF $F_2$. This process continues up to $n-r+1$ components with the common DF $F_r$ work until the $r$ -th failure occurs at time $x_r$ and hence the whole system fails. The mentioned system is called *sequential* $r$ *-out-of-* $n$ *system* and the system lifetime is then $r$ -th component failure time, denoted by $X^*_{(r)}$. In the literature, $(X^*_{(1)},\cdots,X^*_{(n)})$ is called Sequential order statistics. Examples of such phenomena include automobile industries, gas and oil transmission pipelines, etc. Statistical inference on the basis of sequential order statistics have been considered in the literature. For example, Bedbur [2] obtained the uniformly most powerful unbiased test under a conditional proportional hazard rates (CPHR) model via a decision-theoretic approach. To describe the CPHR model, let $\bar{F}_j(t)=\bar{F}_0^{\,\alpha_j}(t)$, for $j=1,\cdots,r$, where $F_0(t)$ is a given underlying DF. In this case, the hazard rate function of the DF $F_j$, defined by $h_j(t)=f_j(t)/\bar{F}_j(t)$ for $t>0$ and $j=1,\cdots,n$, is proportional to the hazard rate function of the baseline DF $F_0$, i.e. $h_j(t)=\alpha_j h_0(t)$. See also, Schenk *et al.*[11], Burkschat *et al.* [5], Cramer *et al.*[6], Beutner *et al.*[4] and references therein. In this paper, we consider the DF $F_0(t)$ be the exponential distribution denoted by $Exp(\pm)$ , i.e.

$$F_0(x;\pm)=1-\exp\left\{-\left(\frac{x}{\pm}\right)\right\},\ x>0,\ \pm>0. \qquad (1)$$

The problem of hypotheses testing for exponential populations on the basis of multiple Sequential order statistics samples under the CPHR model via a Bayesian approach is studied.

To do this, denote the available data by

$$\mathbf{x}=\begin{bmatrix} x_{11} & \cdots & x_{1r} \\ \vdots & \ddots & \vdots \\ x_{s1} & \cdots & x_{sr} \end{bmatrix}, \qquad (2)$$

where the $i$ -th row of the matrix $\mathbf{x}$ in Equation (2) stands for the SOS sample coming from the $i$ -th population. In general, the LF of the available data given by Equation (2) is then