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Italian domination in digraphs

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Abstract

If $D = (V, A)$ be a finite and simple digraph, then an Italian dominating function (IDF) on a digraph D is a labeling $f : V(D) \to \{0, 1, 2\}$ such that for every vertex $v \in V$ with $f(v) = 0$, either v has a in-neighbor with label 2, or v has least two in-neighbor with label 1. The weight of an Italian dominating function f is the value $\omega(f) = \sum_{v \in V} f(v)$. The Italian domination number of a digraph D, denoted by $\gamma_I(D)$, equals the minimum weight of an Italian dominating function on D. In this paper we initiate the study of Italian dominating in digraphs and we present some bounds on $\gamma_I(D)$.
Keywords: Roman domination, Italian domination, 2-rainbow domination number.

1 Introduction

Let D be a finite simple digraph with vertex set $V(D) = V$ and arc set $A(D) = A$. A digraph without directed cycles of length 2 is an *oriented graph*. The order $n = n(D)$ of a digraph D is the number of its vertices. We write $d_D^+(v)$ for the outdegree of a vertex v and $d_D^-(v)$ for its indegree. The minimum and maximum indegree and minimum and maximum outdegree of D are denoted by $\delta^- = \delta^-(D)$, $\Delta^- = \Delta^-(D)$, $\delta^+ = \delta^+(D)$ and $\Delta^+ = \Delta^+(D)$ respectively. If uv is an arc of D, then we also write $u \to v$, and we say that v is an out-neighbor of u and u is an in-neighbor of v. For a vertex v of a digraph D, we denote the set of in-neighbors and out-neighbors of v by $N^-(v) = N_D^-(v)$ and $N^+(v) = N_D^+(v)$, respectively. Let $N^-[v] = N_D^-(v) \cup \{v\}$ and $N^+[v] = N_D^+(v) \cup \{v\}$. For $S \subseteq V(D)$, we define $N^+[S] = \bigcup_{v \in S} N^+[v]$. Consult [5] and [10] for the notation and terminology which are not defined here. A vertex v dominates all vertices in $N^+[v]$. A subset S of vertices of D is a *dominating set* if S dominates $V(D)$. The domination number $\gamma(D)$ is the minimum cardinality of a dominating set of D. The domination number of digraphs was introduced by Chartrand, Harary and Yue [4] as the out-domination number and has been studied by several authors (see, for example [8] and [11]). A *Roman dominating function* (RDF) on a digraph D is a function $f : V \to \{0, 1, 2\}$ satisfying the condition that every vertex v for which $f(v) = 0$ has an in-neighbor u for which $f(u) = 2$. The weight of an RDF f is the value $\omega(f) = \sum_{v \in V} f(v)$. The *Roman domination number* of a digraph D, denoted by $\gamma_R(D)$, equals the minimum weight of an RDF on D. The Roman domination number in digraphs was introduced by Kamaraj and Jakkammal [6] and has been studied in [9]. A Roman dominating function $f : V \to \{0, 1, 2\}$ can be represented by the ordered partition (V_0, V_1, V_2)(or(V_0^f, V_1^f, V_2^f) to refer f) of V, where $V_i = \{v \in V \mid f(v) = i\}$. The 2-rainbow domination number in digraphs was introduced by Amjadi et al. and has been studied in [1]. A 2-rainbow dominating function (2RDF) of a digraph D is a function f from the vertex set V to the set of all subsets of the set $\{1, 2\}$ such that for any vertex $v \in V$ with $f(v) = \emptyset$ the condition $\bigcup_{u \in N^-(v)} f(u) = \{1, 2\}$ is fulfilled, where $N^-(v)$ is the set of in-neighbors of v. The weight of a 2RDF f is the value $\omega(f) = \sum_{v \in V} |f(v)|$. The 2-rainbow domination number of a digraph D, denoted by $\gamma_{2r}(D)$, is the minimum weight of a 2RDF of D.

An Italian dominating function on a digraph D is a labeling $f : V \to \{0, 1, 2\}$ such that every vertex $v \in V$ with $f(v) = 0$, either v has a in-neighbor with label 2, or v has least two in-neighbor with label 1. The weight of an Italian dominating function f is the value $\omega(f) = \sum_{v \in V} f(v)$. The Italian domination number of a digraph D, denoted by $\gamma_I(D)$, equals the minimum weight of an Italian dominating function on D. An Italian dominating function (IDF) $f : V \to \{0, 1, 2\}$ can be represented by the or-