On location-domination in graphs

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Abstract

A subset S of the vertex set $V(G)$ of a graph G is a dominating set if each vertex $v \in V(G) - S$ is adjacent to at least one vertex of S. The minimum dominating set of graph G is called domination number of G and denoted by $\gamma(G)$. A dominating set S is defined to be locating if for any two distinct vertices v and u in $V(G) - S$, $N_G(v) \cap S \neq N_G(u) \cap S$, where $N_G(v)$ (respectively $N_G(u)$) is the set of vertices adjacent to v (respectively u) in G. The *location-domination number* of G, denoted by $\gamma_l(G)$ is the minimum cardinality of a locating-dominating set of G. The *bondage number*, $\mathrm{b}(G)$ of a graph G is the cardinality of the smallest set E' of edges for which $\gamma(G - E') > \gamma(G)$. The *subdivision number*, $\mathrm{sd}_\gamma(G)$ of a graph G is the minimum number of edges that must be subdivided (where each edge in G can be subdivided at most once) in order to increase the domination number of G. The *location-domination bondage number*, $b_l(G)$ of a graph G is the cardinality of the smallest set E' of edges for which $\gamma_l(G - E') > \gamma_l(G)$. The *location-domination subdivision number*, $\mathrm{sd}_{\gamma_l}(G)$ of a graph G is the minimum number of edges that must be subdivided (where each edge in G can be subdivided at most once) in order to increase the location-domination number of G. In this paper we establish the locating bondage number and location-domination subdivision number of paths and cycles.

Keywords: domination, location-domination, bondage number, location-domination bondage number, subdivision number, location-domination subdivision number.

1 Introduction

Let G be a finite simple graph with vertex set $V(G) = V$ and edge set $E(G) = E$. The *order* $n = n(G)$ of a graph G is the number of its vertices. The minimum and maximum degree of G are denoted by $\delta = \delta(G)$, $\Delta = \Delta(G)$, respectively. The *open neighborhood* of a vertex $v \in V$ is $N_G(v) = \{u \in V | uv \in E\}$ and the *close neighborhood* is $N_G[v] = \{u \in V | uv \in E\} \cup \{v\}$. The *degree* of a vertex $v \in V$ is $\deg_G(v) = |N(v)|$. The distance between vertices $v, w \in V$ is denoted by $d_G(v, w)$. We write P_n for the *path* of order n and C_n for the *cycle* of order n. The reader is referred to [?] for the notation and terminology which are not defined here.

A vertex set S of a graph G is a dominating set if each vertex of G either belongs to S or is adjacent to a vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality over all dominating sets of G. A dominating set of G of cardinality $\gamma(G)$ is called a $\gamma(G)$-set. The literature on the subject of domination parameters in graphs up to the year 1997 has been surveyed and detailed in the two books [?] and [?].

A set $S \subseteq V$ is a *locating set* if every vertex is uniquely determined by its vector of distances to the vertices in S. The *location number* of G, denoted by $\beta(G)$ is the minimum cardinality of a locating set of G [?, ?, ?].

For safeguards analysis of a facility, such as a fire protection study of nuclear power plants [?, ?], the facility can be modeled by a graph or network. A vertex can represent a room, hallway, stairwell, courtyard, etc. One primary function of a safeguards system is detection of some object, such as detection a fire or detection of an intruder such as a saboteur. Suppose we want to locate a detection device at a vertex and the device can supply three outputs: there is an object at that vertex, there is an object on one of the vertices adjacent to that vertex (but which adjacent vertex can not be specified), or no object occupies that vertex or any adjacent one. What is necessary is to determine a collection of locations at which to place the detection device so that if an object is placed at any vertex in the graph, then the object can be detected and its position