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A new approach to synchronize fractional–order cancer model with NSFD scheme

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Abstract

This paper presents the synchronization between a pair of cancer chaotic systems and fractional–order time derivative using active control method. The fractional derivative is described in Grünwald sense. Numerical simulation results show that the method is effective and reliable for synchronizing the fractional–order chaotic systems while it allows the system to remain in the chaotic state.

1 Introduction

The use of fractional–orders differential and integral operators in mathematical models has become increasingly widespread in recent years [4, 7, 8, 9, 10]. These models have been the focus of many studies due to their frequent appearance in various applications in fluid mechanics, economic, viscoelasticity, biology, physics and engineering [6]. Recently, most of the dynamical systems based on the integer–order calculus have been modified into the fractional–order domain due to the extra degrees of freedom and the flexibility which can be used to precisely fit the experimental data much better than the integer–order modeling. Chaotic systems have a profound effect on its numerical solutions and are highly sensitive to time step sizes. It will be beneficial to find a reliable analytical tool to test its long–term accuracy and efficiency. Extensive numerical work has been carried out in order to understand chaos in dynamical systems. The chaotic synchronization have applied in many different fields since have been discovered, such as biological systems, structural engineering and ecological models [5]. In this paper, we employ the active control technique to study the synchronization of commensurate fractional–order cancer model. The stability theory of linear commensurate fractional differential equations is utilized to derive the stability of error system which guarantees that the two systems reach complete synchronization. The numerical solution of the master, slave and error systems using nonstandard finite difference scheme (NSFD) scheme given by Mickens [2, 3] are proposed.

2 Preliminaries

The initial foundation of the NSFD schemes came from the exact finite difference schemes. NSFD schemes were firstly proposed by Mickens [2, 3] for either ordinary differential equations (ODEs) or partial differential equations (PDEs) and, successively, their use have been investigated in several fields. Regarding the positivity and boundedness of solutions, the NSFD schemes have a better performance over the standard finite difference schemes, due to its flexibility to construct a NSFD scheme that can preserve certain properties and structures, which are obeyed by the original equations. The advantages of NSFD schemes have been shown in many numerical applications. Zibaei and Namjoo [7, 8, 9] studied NSFD schemes for the numerical solution of biological models. This class of schemes and their formulations center on two issues. First, how should discrete representations for derivatives be determined, and second, what are the proper forms to be used for nonlinear terms. The forward Euler method is one of the simplest discretization schemes. In this method the derivative term $\frac{dy}{dt}$ is replaced by $\frac{y(t+h)-y(t)}{h}$, where h is the step size. However, in the Mickens schemes this term is replaced by $\frac{y(t+h)-y(t)}{\phi(h)}$ where $\phi(h)$ is an increasing continuous function of h, and the denominator function $\phi(h)$ satisfies the following conditions

$$\phi(h) = h + O(h^2), \qquad 0 < \phi(h) < 1, \qquad h \to 0.$$