Polynomial approximations for the solution of two-dimensional linear Volterra integral equations

A. Darijani*, Academic member of Dept. of Math. Higher Education complex of Bam, Kerman, Iran,

Abstract: In this paper, the residual function minimization approach is applied for the approximation of solution of two-dimensional Volterra integral equations. In this method, the solutions of the two-dimensional integral equations are approximated as a linear combination of polynomials. The unknown parameters of an approximate solution are obtained based on minimization of the residual function. Existence and convergence of these approximate solutions are investigated. For to confirm the efficiency and accuracy of the proposed method, some numerical examples are presented.
Keywords: Two-dimensional Integral Equations, Minimization, Residual Function, Gronwall Inequality.

1 INTRODUCTION

There are many problems in applied sciences, such as mechanical engineering and the physical sciences, that give rise to two-dimensional Volterra integral equations, of the following form

$$\begin{aligned} u(x,y) = \int_0^x k_1(x,y,s)u(s,y)ds + \\ \int_0^y k_2(x,y,t)u(x,t)dt + \\ \int_0^x \int_0^y k_3(x,y,t,s)u(s,t)dtds + g(x,y), \end{aligned} \tag{1}$$

where functions $k_1(x,y,s), k_2(x,y,t), k_3(x,y,s,t)$ and $g(x,y)$ are known and $u(x,y)$ is an unknown scalar valued function defined on $\Omega = [0,1] \times [0,1]$ to be determined. Several numerical methods for approximating the solution of the Equation (1) have been presented. Many researchers used polynomials[1, 2], Spline functions[3], Radial basis functions[4] and Wavelets[5] for the approximation of solution (1). these methods are based on the appropriate linear combinations of some basic functions such as Chebyshev polynomials, Legendre polynomials,Taylor polynomials,cubic Spline functions, and wavelets. In present work, the solution of the given integral equation is represented as a linear combination of a set of linearly independent functions. In addition, the calculation method of the corresponding coefficients of these functions is discussed based on minimizing of the related residual function. The concept of residual function minimization has been introduced in [6]. In this paper, the basic idea[6, 7] has been developed and applied to two-dimensional integral equations. In fact, the solution of equation (1) is approximated by a linear combination of some basic polynomials. The polynomial approximations are obtained based on the minimization of the norm of the residual function. Based on the proposed method, the problem of solving a two-dimensional integral equation is converted to a minimization problem.

*Corresponding Author