# Graded Weakly Primary Submodules

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## Abstract

Let $G$ be an arbitrary group with identity $e$, $R$ be a commutative ring with $1 \neq 0$ and $M$ be a unitary $R$-module. Weakly prime ideals and weakly prime submodules have been introduced and studied in [1] and [5], respectively. Here we study the graded weakly primary submodules of a $G$-graded $R$-module $M$. Some properties of graded weakly primary submodules are considered.

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## 1 Introduction and preliminaries

Weakly primary ideals in a commutative ring with non-zero identity have been introduced and studied in [2]. In this paper, we study graded weakly primary submodules of graded modules over graded commutative rings and the structures of their homogeneous components.

Now, we introduce some notations and terminologies. Let $G$ be a group with identity $e$ and $R$ be a commutative ring. Then $R$ is a $G$-graded ring if there exist additive subgroups $R_g$ of $R$ such that $R = \oplus_{g\in G} R_g$ and $R_{g_1}R_{g_2} \subseteq R_{g_1g_2}$ for all $g_1, g_2 \in G$. The elements of $R_g$ are called homogeneous of degree $g$. Let $r \in R$, then $r$ can be written uniquely as $\sum_{g\in G} r_g$, where $r_g$ is the component of $r$ in $R_g$ . We write $h(R) = \cup_{g\in G} R_g$. Also, $R_e$ is a subring of $R$ with $1_R \in R_e$.

We consider $suppR = \{g \in G : R_g \neq 0\}$. Let $I$ be an ideal of $R$. For $g \in G$, let $I_g = I \cap R_g$. Then $I$ is a graded ideal of $R$ if $I = \oplus_{g\in G} I_g$. In this case, $I_g$ is called the $g$-component of $I$ for $g \in G$.

The graded radical of a graded ideal $I$ of $R$, denoted by $Grad(I)$, is the set of all $x \in R$ such that for each $g \in G$ there exists $n_g > 0$ with $x_g^{n_g} \in I$. Note that, if $r$ is a homogeneous element of $R$, then $r \in Grad(I)$ if and only if $r^n \in I$ for some positive integer $n$.

Let $I$ be a graded ideal of $R$ and $x \in G$. The set $xrad(I) = \{a \in R_x : a^n \in I, \text{for some positive integer n}\}$ is a subgroup of $R_x$. Clearly, $I_x \subseteq xrad(I)$ and If $r \in R_x$ with $r \in Grad(I)$, then $r \in xrad(I)$.

Let $R$ be a $G$-graded ring and $M$ be an $R$-module. We say that $M$ is a $G$-graded R-module (or graded $R$-module), if there exist subgroups $M_g$ of $M$ such that $M = \oplus_{g\in G} M_g$ (as abelian groups) and $R_{g_1}M_{g_2} \subseteq M_{g_1g_2}$ for all $g_1, g_2 \in G$. We write $h(M) = \cup_{g\in G} M_g$ and the elements of $h(M)$ are called homogeneous.

Let $M = \oplus_{g\in G} M_g$ be a graded $R$-module and $N$ be a submodule of $M$. Then $N$ is called a graded submodule of $M$ if $N = \oplus_{g\in G} N_g$, where $N_g = N \cap M_g$, for $g \in G$. Also, $N_g$ is called the $g$-component of $N$. Moreover, $\frac{M}{N}$ is a $G$-graded $R$-module with $g$-component $(\frac{M}{N})_g = \frac{(M_g+N)}{N}$, where $g \in G$, see [3].

A proper graded submodule $N$ of a graded module $M$ is said to be graded prime (resp., graded weakly prime) submodule if whenever $r \in h(R)$ and $m \in h(M)$ together with $rm \in N$ (resp., $0 \neq rm \in$

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