

Original Article

Comparing parameter choice methods for regularization
of ill-posed problemsFrank Bauer^a, Mark A. Lukas^{b,*}^a Fuzzy Logic Laboratorium Linz-Hagenberg, University of Linz, Softwarepark 21, 4232 Hagenberg, Austria^b Mathematics and Statistics, Murdoch University, South Street, Murdoch, WA 6150, Australia

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Abstract

In the literature on regularization, many different parameter choice methods have been proposed in both deterministic and stochastic settings. However, based on the available information, it is not always easy to know how well a particular method will perform in a given situation and how it compares to other methods. This paper reviews most of the existing parameter choice methods, and evaluates and compares them in a large simulation study for spectral cut-off and Tikhonov regularization. The test cases cover a wide range of linear inverse problems with both white and colored stochastic noise. The results show some marked differences between the methods, in particular, in their stability with respect to the noise and its type. We conclude with a table of properties of the methods and a summary of the simulation results, from which we identify the best methods.

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1. Introduction

Consider the linear inverse problem

$$Ax = y, \tag{1}$$

where A is a linear compact operator mapping between two separable Hilbert spaces $\mathcal{X}$ and $\mathcal{Y}$. In practical situations, only a noisy version y^δ of y is available as data. Because of the compactness of A , solving (1) for x is unstable, and one needs to regularize the problem to obtain a reasonable approximate solution [41].

The two most popular regularization methods are spectral cut-off regularization (also called truncated singular value decomposition) and Tikhonov regularization (also called ridge regression or Wiener filtering in certain contexts). For both regularization methods, the choice of the regularization parameter is crucial to yield a good solution.

Over the last four decades, many quite different methods for choosing this parameter have been proposed. Most of these methods have been developed with some analytical justification, but this is usually confined to a particular

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