Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation 81 (2011) 1006–1016

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

On the error in approximating stability spectra for discrete dynamical systems☆

Melissa Menning, Erik S. Van Vleck *

Department of Mathematics, University of Kansas, Lawrence, Kansas 66045, United States

Available online 21 October 2010

Abstract

In this work, we develop a perturbation theory for the approximation of stability spectra (Lyapunov exponents and Sacker-Sell spectrum) for discrete time dynamical systems. The approach is based upon *QR* based methods that transform linear time varying discrete time systems to an upper triangular form that allows for extraction of the stability spectra. We focus on nonlinear planar maps and show how shadowing type error results can be combined with perturbation bounds for stability spectra. The utility of our results is illustrated with numerical experiments for the Hénon and standard maps.

Keywords: Discrete dynamical systems; *QR* methods; Lyapunov exponents; Integral separation; Planar maps

1. Introduction

In this paper we adapt and extend the error analysis for approximation of stability spectra, Lyapunov exponents and Sacker-Sell spectrum, developed in [6–10,20] to discrete dynamical systems. We consider nonlinear mappings of the form

$$x_{n+1} = f_n(x_n) \tag{1}$$

and the subsequent linearized equation

$$u_{n+1} = Df_n(x_n)u_n \equiv A_n u_n \tag{2}$$

Our contribution in this paper, besides phrasing all assumptions in terms of discrete time dynamical systems, is to make this theory accessible to the large number of discrete time dynamical systems, more carefully examine the form of the error obtained in [7], and provide for a class of planar mapping a way to obtain both a shadowing result and the error in stability spectra (Lyapunov exponents and Sacker-Sell spectrum) simultaneously.

This work builds upon the body of work begun in [6] where the consequences of integral separation was examined in the context of *QR* based methods (writing the fundamental matrix solution as orthogonal times upper triangular with positive diagonal elements) for determining stability spectra. In [9] it was shown that for stable, i.e., continuous with respect to perturbations, Lyapunov exponents and for Sacker-Sell spectrum the *QR* based methods are justified since these stability spectra may be extracted from the diagonal elements of the upper triangular factors. A backward

☆ This work was supported in part under NSF Grants DMS-0513438 and DMS-0812800.
* Corresponding author.
E-mail addresses: menning@math.ku.edu (M. Menning), evanvleck@math.ku.edu (E.S. Van Vleck).

0378-4754/$36.00
doi:10.1016/j.matcom.2010.10.006