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On the deterministic solution of multidimensional parametric models using the Proper Generalized Decomposition

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Abstract

This paper focuses on the efficient solution of models defined in high dimensional spaces. Those models involve numerous numerical challenges because of their associated curse of dimensionality. It is well known that in mesh-based discrete models the complexity (degrees of freedom) scales exponentially with the dimension of the space. Many models encountered in computational science and engineering involve numerous dimensions called configurational coordinates. Some examples are the models encountered in biology making use of the chemical master equation, quantum chemistry involving the solution of the Schrödinger or Dirac equations, kinetic theory descriptions of complex systems based on the solution of the so-called Fokker–Planck equation, stochastic models in which the random variables are included as new coordinates, financial mathematics, etc. This paper revisits the curse of dimensionality and proposes an efficient strategy for circumventing such challenging issue. This strategy, based on the use of a Proper Generalized Decomposition, is specially well suited to treat the multidimensional parametric equations.

Keywords: Multidimensional models; Curse of dimensionality; Parametric models; Proper Generalized Decompositions; Separated representations

1. Introduction: revisiting the Proper Generalized Decomposition

The efficient solution of complex models involving an impressive number of degrees of freedom could be addressed by performing high performance computing (in general making use of parallel computing platforms) or by speeding up the calculation by using preconditioning, domain decomposition, etc.

In the case of transient models the use of model reduction can alleviate significantly the solution procedure. The main ingredient of model reduction techniques based on the use of proper orthogonal decompositions – POD – consists of extracting a reduced number of functions able to represent the whole time evolution of the solution, that could be then used to make-up a reduced model. This extraction can be performed by invoking the POD. The reduced model can be then used for solving a similar model, i.e. a model slightly different to the one that served to extract the reduced approximation basis, or for solving the original model in a time interval larger than the one that served for constructing the reduced basis. The main issue in this procedure consists in the evaluation of the reduced basis quality when it applies

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