

Original article

Endogenous cycles in discontinuous growth models

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Abstract

In this paper we consider a discontinuous one-dimensional piecewise linear model describing a neoclassical growth model. These kind of maps are widely used in the applied context. We determine the analytical expressions of border collision bifurcation curves, responsible for the observed dynamics, which consists of attracting cycles of any period and of quasiperiodic trajectories in exceptional cases.

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1. Introduction

Bifurcations in a piecewise-smooth system are quite different from those occurring in a smooth one. It is nowadays well known that in smooth systems the dynamics may evolve from a regular dynamic behavior to a complex one via a sequence of bifurcations (as, for example, routes to chaos via Feigenbaum cascades of period doubling bifurcations), while in piecewise smooth systems *border-collision bifurcations* (BCB for short) may occur. Of interest in this paper are piecewise-linear systems for which only BCB occur. Border-collision refers to any contact between an invariant set of a map with the border of its region of definition, and this generally gives rise to a *bifurcation*. The term *border-collision bifurcation* was used for the first time by Nusse and Yorke [33] (see also [34]) and it is now widely used in this context (i.e. for piecewise smooth maps). These bifurcations have been widely studied in recent years, mainly because of their relevant applications in physics and engineering [1–3,6,13,14,45]. However, the study and description of such border collision bifurcations started a long time ago. We remark that the bifurcations associated with piecewise smooth maps are studied in [27–29,31,32] even if the bifurcations were not called of border-collision. In particular, in [14] some results by Feigin are republished, these were already printed in 1978¹ (but not known widely). We may also go further back, citing the works by Leonov in the 60s, [25,26]. In his works, Leonov described several bifurcations, giving a recurrence relation to find the analytic expression of the family of bifurcations occurring in a one-dimensional piecewise linear map with one discontinuity point, which also is still mainly unknown. Some of his results have been

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¹ It is worth noticing that the clear and simple analysis performed by Feigen in 1978 is the first one for *n-dimensional piecewise linear continuous maps*, with $n > 1$.