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On the convergence of quasi-random sampling/importance resampling

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Abstract

This article discusses the general problem of generating representative point sets from a distribution known up to a multiplicative constant. The sampling/importance resampling (SIR) algorithm is known to be useful in this context. Moreover, the quasi-random sampling/importance resampling (QSIR) scheme, based on quasi-Monte Carlo methods, is a more recent modification of the SIR algorithm and was empirically shown to have better convergence. By making use of quasi-Monte Carlo theory, we derive upper bounds for the error of the QSIR scheme.

Keywords: Sampling/importance resampling; Weighted bootstrap; Quasi-Monte Carlo methods; Bayesian inference

1. Introduction

A frequently occurring problem in many fields of science is the generation of samples from a given density. Many methods exist to solve this problem, see for example the monographs of Gentle [5] or Devroye [3]. One of the possible methods uses the inverse of the cumulative distribution function to generate the samples. We will refer to this method as the inverse CDF method. However, when the density is only known up to a multiplicative constant, the inverse CDF method cannot be used.

A method to obtain samples that have an approximate distribution with density $h(\boldsymbol{\theta})$ was suggested by Rubin in [12,13] and is called *sampling /importance resampling* (SIR). It has the advantage that the normalizing constant of the target density is not needed, so instead of the density $h(\boldsymbol{\theta})$ any nonnegative proportional function $ch(\boldsymbol{\theta})$ can be used.

Two modifications of the SIR algorithm were recently proposed in [11] but not supported by theory. The modified algorithms are based on low-discrepancy point sets and sequences and it is empirically shown that more representative samples can be obtained using the modified algorithms. In this article, we complement the empirical results from [11] with convergence proofs. The proofs are based on quasi-Monte Carlo theory.

The outline of this article is as follows: in Section 2 we recall concepts from quasi-Monte Carlo theory that are useful for the rest of this paper. The ideas behind the SIR algorithm and its recent QSIR variant are given in Section 3. In Section 4, we prove convergence for the QSIR algorithm. Numerical examples are presented in Section 5 and we conclude in Section 6.

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