

Original article

A mathematical model of atmospheric retention of man-made CO₂ emissions

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Abstract

Rust and Thijsse have previously shown that changes in global annual average temperature anomalies $T(t_i)$ compiled by the Climatic Research Unit vary linearly with atmospheric CO₂ concentrations $c(t_i)$. The $c(t_i)$ can be related to man-made CO₂ emissions $F(t_i)$ by a linear regression model whose solution vector gives the unknown retention fractions $\gamma(t_i)$ of the $F(t_i)$ in the atmosphere. Gaps in the $c(t_i)$ record make the system underdetermined, but the constraints $0 \leq \gamma(t_i) \leq 1$ make estimation tractable. The $\gamma(t_i)$ are estimated by two methods: (1) assuming a finite harmonic expansion for $\gamma(t)$, and (2) using a constrained least squares algorithm to compute average values of $\gamma(t)$ on suitably chosen time subintervals. The final result is an estimate of $\gamma(t)$ with enough accuracy to establish the connection between emissions from fossil fuel use and land use changes and increases in global average annual temperature anomalies.

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1. Atmospheric CO₂ and global temperatures

Throughout this paper the terms *warming* and *global warming* will be used to refer to the well documented [8] increases in global average annual temperature anomalies from 1850 to the present. Uncertainties in global climate models are often used to cast doubt on the reality of global warming, but complicated climate models are not needed to establish that reality. Rust and Thijsse [14,16] have used the measurements of global average temperatures and of atmospheric carbon dioxide concentrations to confirm that the warming is real and that it is principally caused by the increasing CO₂ concentrations.

The plot in Fig. 1 gives the record of atmospheric CO₂ concentrations obtained by combining atmospheric measurements (1958–2004) at the South Pole [9] with reconstructions (1647–1978) from Antarctic ice cores [4,11]. Although the latter display larger random variations than the former, the two records are consistent in the years where they overlap. The horizontal dashed line at $c_0 = 277.04$ ppmv is the preindustrial CO₂ concentration estimated by averaging the ice-core measurements for 1647–1764. The solid curve is a cubic regression spline fit obtained from Thijsse's *spline2* program [16] which chooses the number and locations of the knots to give an optimal separation of signal from noise. It will be used in the following as the functional representation $c(t)$ of the concentration measurements.

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