

Original article

Complex dynamics associated with the appearance/disappearance of invariant closed curves

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Abstract

In this paper we study some global bifurcations arising in a heterogeneous financial model with fundamentalists and imitators. Such bifurcations which cause the appearance and disappearance of closed invariant curves (attracting or repelling) involve the stable and unstable sets of a saddle cycle with consequent changes in their dynamic behavior. Numerical investigations show that the transition between two qualitatively different regimes are characterized by the occurrence of homoclinic tangles with chaotic dynamics.

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1. Introduction

The bifurcation mechanisms involved in the appearance (disappearance) of invariant closed curves play a crucial role in the development of complex pattern behaviors in nonlinear maps which arise as discrete-time models of economic dynamical systems. Thus, it is not surprising if in the past decades the Neimark–Sacker bifurcation has become a frequently used tool in the study of nonlinear economic dynamical systems (see [8,21] and references therein). This is due, for instance, to the fact that repelling closed curves may bound the basin of attraction of some stable steady state determining the so-called “corridor stability” of the equilibrium (see, among others [10,19,25,26]). Moreover, the existence of an attracting closed curve may explain the endogenous fluctuations of the economy under scrutiny, corresponding to quasi-periodic or periodic long run behaviors (see [6,7,15]). Recently, contributions to this topic have been given in [1,3,4,11,14,16,28] where the study of the bifurcations are faced both from a mathematical and an economic point of view. In mathematical literature, some bifurcations involving closed invariant curves in discrete dynamical systems have been described in the 1970s and resumed in the books [17,31,32]. In the present decade a new concern is born on this topic (see [2,5,22–24] among the others) suggesting that many features require major studies.

As shown in [1–4], in some classes of maps a typical mechanism related to the appearance/disappearance of closed invariant curves is associated with a *saddle-connection*, that is, a closed connection (or *homoclinic loop*) among the periodic points of a saddle cycle which occurs when a branch of the stable set of the saddle merges with an unstable

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