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GARCH dependence in extreme value models with Bayesian inference

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Abstract

Extreme value methods are widely used in financial applications such as risk analysis, forecasting and pricing models. One of the challenges with their application in finance is accounting for the temporal dependence between the observations, for example the stylised fact that financial time series exhibit volatility clustering. Various approaches have been proposed to capture the dependence. Commonly a two-stage approach is taken, where the volatility dependence is removed using a volatility model like a GARCH (or one of its many incarnations) followed by application of standard extreme value models to the assumed independent residual innovations.

This study examines an alternative one stage approach, which makes parameter estimation and accounting for the associated uncertainties more straightforward than the two-stage approach. The location and scale parameters of the extreme value distribution are defined to follow a conditional autoregressive heteroscedasticity process. Essentially, the model implements GARCH volatility via the extreme value model parameters. Bayesian inference is used and implemented via Markov chain Monte Carlo, to permit all sources of uncertainty to be accounted for. The model is applied to both simulated and empirical data to demonstrate performance in extrapolating the extreme quantiles and quantifying the associated uncertainty.

Keywords: Extreme values; Dependence; Bayesian inference; GARCH

1. Introduction

Modelling the tails of distributions is important in many fields, such as environmental sciences, hydrology, insurance, finance where the risk of unusually large or small events are of interest. Extreme value theory considers the distributional behaviour of the extremes of random variables, towards extrapolating the stochastic dynamics of a process to states with small chances of realization and typically beyond the range of observed data. It is always challenging to justify the form of extreme models and to estimate parameters due to the inherent sparsity of observations in the tails. Hence, asymptotically justified parametric models are typically used to represent the data generating process, which can provide the reliable extrapolations required in such applications.

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