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Fundamental matrix solutions of piecewise smooth differential systems

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Abstract

We consider the fundamental matrix solution associated to piecewise smooth differential systems of Filippov type, in which the vector field varies discontinuously as solution trajectories reach one or more surfaces. We review the cases of transversal intersection and of sliding motion on one surface. We also consider the case when sliding motion takes place on the intersection of two or more surfaces. Numerical results are also given.

Keywords: Piecewise smooth dynamical systems; Filippov; Sliding modes; Fundamental matrix

1. Introduction and background

Our purpose in this paper is to survey definitions and properties of the fundamental matrix solution associated to piecewise smooth differential equations. Many of the results we give are available in the literature, but are not all readily available. Moreover, some of the extensions we consider herein, such as when there is sliding motion on intersection of surfaces, appear new.

We study differential equations with discontinuous right-hand side, and more precisely equations in which the right-hand side changes discontinuously as one or more surfaces are crossed. These surfaces are called *discontinuity or switching surfaces* and the systems under study are called *piecewise smooth dynamical systems* in Ref. [9].

We only consider the case of continuous solutions for an initial value problem associated to these systems (e.g., "impact" systems are not treated). The interesting case is what happens to a trajectory when it reaches a switching surface. Loosely speaking, there are two things which can occur: we may cross the surface, or we may stay on it, in which case Filippov's construction (see [11]) will define a vector field on the sliding surface and the motion will be called *sliding mode*.

Piecewise smooth differential equations appear pervasively in applications of various nature (see, e.g. [5,14,15,17,24]); for a significative sample of references in the context of control, see e.g. [25–27]; in the context of biological systems, see e.g. [7,8,15,24]; in the context of mechanical systems, see e.g. [13,20,21]; see the classical references [4,11,26,27] and the recent book [9] for a theoretical introduction to these systems, and finally see the recent book [1] for a review of numerical methods used on these systems.

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