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Original article

Singularly perturbed advection–diffusion–reaction problems: Comparison of operator-fitted methods

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Abstract

We compare classical methods for singular perturbation problems, such as El-Mistikawy and Werle scheme and its modifications, to exponential spline collocation schemes. We discuss subtle differences that exist in applying this method to one dimensional reaction–diffusion problems and advection–diffusion problems. For pure advection–diffusion problems, exponential tension spline collocation is less capable of capturing only one boundary layer, which happens when no reaction term is present. Thus the existing collocation scheme in which the approximate solution is a projection to the space piecewisely spanned by $\{1, x, \exp(\pm px)\}$ is inferior to the generalization of El-Mistikawy and Werle method proposed by Ramos. We show how to remedy this situation by considering projections to spaces locally spanned by $\{1, x, x^2, \exp(px)\}$, where $p>0$ is a tension parameter. Next, we exploit a unique feature of collocation methods, that is, the existence of special collocation points which yield better global convergence rates and double the convergence order at the knots.

MSC: 34D15; 35K57; 65D07; 65N35

Keywords: Singular perturbations; Boundary layers; Collocation; Difference schemes; Exponential splines

1. Introduction

We compare some popular methods for solving a singularly perturbed boundary value problem

$$-\varepsilon u''(x) + a(x)u'(x) + b(x)u(x) = f(x), \quad 0 < x < 1, \tag{1}$$

$$u(0) = \alpha, \quad u(1) = \beta, \tag{2}$$

where $0<\varepsilon \ll 1$ and $a(x)$, $b(x)$, $f(x)$ are sufficiently smooth. The usual assumptions for this type of problems are $a(x)>a^*>0$, $\varepsilon \ll a^*$ and $b(x) \geq 0$. In this paper we also consider a case where the advection coefficient $a(x)$ changes

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