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An Unfitted Discontinuous Galerkin method for pore-scale simulations of solute transport

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Abstract

For the simulation of transport processes in porous media effective parameters for the physical processes on the target scale are required. Numerical upscaling, as well as multiscale approaches can help where experiments are not possible, or hard to conduct.

In 2009, Bastian and Engwer proposed an Unfitted Discontinuous Galerkin (UDG) method for solving PDEs in complex domains, e.g. on the pore scale. We apply this method to a parabolic test problem. Convergence studies show the expected second order convergence. As an application example solute transport in a porous medium at the pore scale is simulated.

Macroscopic breakthrough curves are computed using direct simulations. The method allows finite element meshes which are significantly coarser then those required by standard conforming finite element approaches. Thus it is possible to obtain reliable numerical results for macroscopic parameter already for a relatively coarse grid.

Keywords: Discontinuous Galerkin method; Higher order; Unfitted finite elements; Numerical upscaling; Flows in porous media; Convection–diffusion-problems; Parabolic equations

1. Introduction

1.1. Motivation

The correct prediction of solute transport in porous media is important for many different applications like groundwater contamination and remediation. On the continuum scale solute transport is commonly simulated using the convection–dispersion equation. Two major difficulties arise in this approach: (i) It is hard to determine the correct dispersion coefficient, which depends not only on the solute itself but also on the porous medium and the flow velocity. (ii) The convection–dispersion equation is only valid in the limit of travel distances which are large enough for the solute to sample all the heterogeneities of the porous medium. On shorter travel distances one often experiences anomalous dispersion with an earlier breakthrough and a long tailing [16,26].

Pore scale modeling of water and solute transport is a helpful tool to derive macroscopic parameters like the dispersion coefficient as well as to study the anomalous dispersion. At the pore scale water transport is described by the (Navier) Stokes equation, solute transport by the convection–diffusion equation. While a plausible microscopic structure was

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