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Original article

Variational trivariate fitting using Worsey–Piper macro elements on tetrahedral partitions☆

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Abstract

In this paper we present a method to obtain a trivariate spline constructed over the Worsey–Piper split corresponding to a tetrahedron. Such spline approximates a set of Lagrangian scattered data by minimizing an "energy functional" which also controls the smoothness of the spline. We give a convergence result and we show some graphical examples.

Keywords: Minimal energy; Worsey–Piper split; Variational spline; Approximation; Smoothing

1. Introduction and preliminaries

In the last years different variational methods based on the minimization of an "energy functional" have been used in CAD and CAGD in the field of the fitting and design of curves and surfaces. Different works in this research area have shown that these methods are useful and efficient (see [1], for instance). The energy functional is usually a sum of two terms: the first measures how well the resulting curve or surface approximates a set of scattered data which may be of Lagrange or Hermite type. The second represents the minimal energy condition, which is usually defined as a linear combination of seminorms in an adequate Sobolev space.

As background of this theory we can mention the discrete smoothing D^m-splines [2,3] and the discrete smoothing variational splines [10], which provide specific examples of variational curves and surfaces. Related to this field of research we can also cite [4] or [5], where a variational approach is developed for approximating non-noisy and noisy data, respectively, by using surfaces constructed over Powell-Sabin triangulations of a given polygonal domain D. In all cases, some smoothing parameters appear in the definition of the minimization functionals. The estimation of the values of these parameters that minimize a given measure is an important problem that has studied by several authors (see [8,9,19–21], for instance).

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