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Original article

On a mathematical model of journal bearing lubrication

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Abstract

We consider the isothermal steady motion of an incompressible fluid whose viscosity depends on the pressure and the shear rate. The system is completed by suitable boundary conditions involving non-homogeneous Dirichlet, Navier's slip and inflow/outflow parts. We prove the existence of weak solutions and show that the resulting level of the pressure is fixed by the boundary conditions. The paper is motivated by the journal bearing lubrication problem and extends the earlier results for homogeneous boundary conditions.

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Keywords: Existence of weak solutions; Incompressible non-Newtonian fluids; Pressure and shear rate dependent viscosity; Inflow/outflow boundary conditions; Filtration boundary conditions

1. Introduction

When mathematically describing the flow of an incompressible viscous fluid, a common hypothesis is that the viscous forces are a linear function of the velocity gradient and are independent of other variables, namely the pressure. Such assumption is inherent with the model of the so-called Newtonian fluid governed by the Navier–Stokes system. A number of generalizations have been made in order to capture phenomena that are observed in various fluids at various operating conditions and cannot be covered by the Newtonian model. In this paper we address two such features: the shear-thinning, where the viscosity decreases with the shear rate, and the pressure-thickening, where it increases with the pressure. It is worth noting that already Stokes [46] recognized that the viscous forces may be independent of the pressure only within a limited range of pressures. Note also that while changes of the viscosity due to the pressure can be severe, density variations can remain insignificant by comparison, so that the assumption of incompressibility is not violated.

There is a particular distinction of the pressure-thickening models, which partly inspired our study. It is a well-known property of the equations describing the motion of incompressible fluids that the pressure is determined to within a

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