Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 81 (2011) 1593–1608
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original article

A mollification regularization method for stable analytic continuation☆

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Received 7 July 2009; received in revised form 7 November 2010; accepted 29 November 2010
Available online 14 December 2010

Abstract

In this paper, we consider an analytic continuation problem on a strip domain with the data given approximately only on the real axis. The Gauss mollification method is proposed to solve this problem. An *a priori* error estimate between the exact solution and its regularized approximation is obtained. Moreover, we also propose a new *a posteriori* parameter choice rule and get a good error estimate. Several numerical examples are provided, which show the method works effectively.

AMS Classification: 30B40; 65J20

Keywords: Analytic continuation; Ill-posed problems; Mollification method; Error estimate; *A posteriori*

1. Introduction

The analytic continuation is a classical problem in complex analysis, which is frequently encountered in many practical applications [6,15,19,22], while the stable numerical analytic continuation is a rather difficult problem. In general, this problem is severely ill-posed. Some theoretical and numerical studies have been devoted to the problem. The earlier works mainly focused on the results of the conditional stability [12,15]. However, it seems that there are few applications of modern theory of regularization methods which have been developed intensively in the last few decades. A simple computer algorithm was given in [6] that is based on the fast Fourier transform. In [2,24], a hypergeometric summation method was used to reconstruct some analytic functions from exponentially spaced samples, where the approximation errors and stability estimates were obtained.

In this paper, we consider the following problem of analytic continuation. Let function $f(z) = f(x + iy)$ be analytic on a strip domain D of the complex plane defined by

$$D := \{z = x + iy \in \mathbb{C} | x \in \mathbb{R}, \quad |y| < y_0, \quad y_0 \text{ is a positive constant}\}, \tag{1.1}$$

where i is the imaginary unit. The data is only given on the real axis, i.e., $f(z)\big|_{y=0} = f(x)$ is known approximately and we will extend f analytically from this data to the whole domain D.

☆ This work is supported by the National Natural Science Foundation of China (Nos. 10671085, 10971089).
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0378-4754/$36.00
doi:10.1016/j.matcom.2010.11.011