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Modelling and simulation of autonomous oscillators with random parameters

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Abstract

We consider periodic problems of autonomous systems of ordinary differential equations or differential algebraic equations. To quantify uncertainties of physical parameters, we introduce random variables in the systems. Phase conditions are required to compute the resulting periodic random process. It follows that the variance of the process depends on the choice of the phase condition. We derive a necessary condition for a random process with a minimal total variance by the calculus of variations. A corresponding numerical method is constructed based on the generalised polynomial chaos. We present numerical simulations of two test examples.

Keywords: Ordinary differential equation; Differential algebraic equation; Uncertainty quantification; Polynomial chaos; Calculus of variations

1. Introduction

Mathematical modelling often yields systems of ordinary differential equations (ODEs) or differential algebraic equations (DAEs). We consider periodic boundary value problems of autonomous systems. Hence the corresponding periods are unknown a priori. Since a continuum of periodic solutions exists, we require phase conditions to isolate a solution.

We assume that some physical parameters of the systems exhibit uncertainties. Thus we replace the parameters by random variables with traditional distributions (uniform, Gaussian, etc.). The periodic solution of the system of ODEs or DAEs becomes a random process. Since different realisations of the parameters imply different periods, the solutions of the systems are transformed to the unit interval in time domain. Again we need phase conditions to determine a particular solution of the stochastic model. Random processes satisfying the same autonomous system but different phase conditions are interconnected by a transformation in time domain. Consequently, they include the same information on the periodic solutions for each realisation of the parameters. However, both the expected value and the variance depend on the choice of the phase condition. A representation of the random process in the phase space does not yield an adequate alternative.

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