

Original article

A two-parameter weighted Lindley distribution and its applications to survival data

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Abstract

A two-parameter weighted Lindley distribution is proposed for modeling survival data. The proposed distribution has the property that the hazard rate (mean residual life) function exhibits bathtub (upside-down bathtub) or increasing (decreasing) shapes. Simulation studies are conducted to investigate the performance of the maximum likelihood estimators and the asymptotic confidence intervals of the parameters. Applications of the proposed model to real survival data are presented.

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1. Introduction

When an investigator records an observation by nature according to certain stochastic model, the recorded observation will not have the original distribution unless every observation is given an equal chance of being recorded. For example, suppose that the original observation x_0 comes from a distribution with probability density function (p.d.f.) $f_0(x_0; \theta_1)$, where θ_1 is a parameter vector, and that observation x is recorded according to a probability re-weighted by a weight function $w(x; \theta_2) > 0$, θ_2 is a new parameter vector, then x comes from a distribution with p.d.f.

$$f(x) = Aw(x; \theta_2)f_0(x; \theta_1)$$

where A is a normalizing constant. Distributions of this type are called *weighted distributions*. The weighted distribution with $w(x; \theta_2) = x$ is called length-bias distribution. Patil and Rao [11] examined some general models leading to weighted distributions and showed how the weight $w(x; \theta_2) = x$ occurs in a natural way in many sampling problems.

The study of weighted distributions is useful for two main purposes: it provides a new understanding of standard distributions, and it provides methods of extending distributions for added flexibility in fitting data.

In this paper we consider a two-parameter weighted Lindley distribution with p.d.f.

$$f(x) = Bx^{c-1}f_0(x; \theta), \quad x > 0, \quad c, \theta > 0,$$

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