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Error analysis for a non-standard class of differential quasi-interpolants☆

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Abstract

Given a B-spline M on $\mathbb{R}^s$, $s \geq 1$ we consider a classical discrete quasi-interpolant Q_d written in the form

$$Q_d f = \sum_{i \in \mathbb{Z}^s} f(i) L(\cdot - i),$$

where $L(x) := \sum_{j \in J} c_j M(x-j)$ for some finite subset $J \subset \mathbb{Z}^s$ and $c_j \in \mathbb{R}$. This fundamental function is determined to produce a quasi-interpolation operator exact on the space of polynomials of maximal total degree included in the space spanned by the integer translates of M, say $\mathbb{P}_m$. By replacing $f(i)$ in the expression defining $Q_d f$ by a modified Taylor polynomial of degree r at i, we derive non-standard differential quasi-interpolants $Q_{D,r}f$ of f satisfying the reproduction property

$$Q_{D,r} p = p, \text{ for all } p \in \mathbb{P}_{m+r}.$$

We fully analyze the quasi-interpolation error $Q_{D,r}f - f$ for $f \in C^{m+2}(\mathbb{R}^s)$, and we get a two term expression for the error. The leading part of that expression involves a function on the sequence $c := (c_j)_{j \in J}$ defining the discrete and the differential quasi-interpolation operators. It measures how well the non-reproduced monomials are approximated, and then we propose a minimization problem based on this function.

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Keywords: B-splines; Differential quasi-interpolants; Discrete quasi-interpolants; Error estimates

1. Introduction

Let M be a s-variate B-spline, i.e. a compactly supported non-negative polynomial piecewise function defined on $\mathbb{R}^s$, $s \geq 1$, normalized by $\sum_{i \in \mathbb{Z}^s} M(\cdot - i) = 1$. Let $\mathcal{S}(M) := \text{span}(M(\cdot - i))_{i \in \mathbb{Z}^s}$ be the cardinal spline space spanned by

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