Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation 81 (2011) 1144–1152

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Original article

A new investigation into regularization techniques for the method of fundamental solutions

Ji Lin [a,b], Wen Chen [a,b,*], Fuzhang Wang [a,b]

[a] *Center for Numerical Simulation Software in Engineering and Sciences, Department of Engineering Mechanics, Hohai University, Box 102, No. 1 XiKang Road, Nanjing, Jiangsu 210098, China*

[b] *State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China*

Received 24 March 2010; received in revised form 21 October 2010; accepted 29 October 2010

Available online 26 November 2010

Abstract

This study examines different regularization approaches to investigate the solution stability of the method of fundamental solutions (MFS). We compare three regularization methods in conjunction with two different regularization parameters to find the optimal stable MFS scheme. Meanwhile, we have investigated the relationship among the condition number, the effective condition number, and the MFS solution accuracy. Numerical results show that the damped singular value decomposition under the parameter choice of the generalized cross-validation performs the best in terms of the MFS stability analysis. We also find that the condition number is a superior criterion to the effective condition number.

Keywords: Method of fundamental solutions; Regularization technique; Regularization parameter; Effective condition number

1. Introduction

In recent years, the method of fundamental solutions has rightfully received a great deal of attention by applied mathematicians and engineers in dealing with a variety of engineering problems [2,4,6–8,14]. This method is a boundary-type meshless method with its merits being integration-free, spectral convergence, and easy to use. Despite these merits, we find that the convergence curves of the MFS numerical solution oscillate when a large number of boundary points are used. This may partially due to the ill-conditioned coefficient matrix [16,17]. Therefore, it is of interest to see if regularization methods, such as the damped singular value decomposition (DSVD), the truncated singular value decomposition (TSVD) or the Tikhonov regularization (TR), can or should be used to mitigate the ill-conditioned effect.

To the best of our knowledge, Kitagawa [15] first used the singular value decomposition (SVD) to deal with the solution of the ill-conditioned MFS equations. Based on the SVD, Ramachandran [21] used an alternative solution procedure to illustrate that the numerical results are extremely accurate and relatively independent of the location of the source points. However, Chen et al. [3] showed that the SVD is not more reliable than Gaussian elimination in

* Corresponding author at: Center for Numerical Simulation Software in Engineering and Sciences, Department of Engineering Mechanics, Hohai University, Box 102, No. 1 XiKang Road, Nanjing, Jiangsu 210098, China.

E-mail address: chenwen@hhu.edu.cn (W. Chen).

0378-4754/$36.00 © 2010 IMACS. Published by Elsevier B.V. All rights reserved.

doi:10.1016/j.matcom.2010.10.030