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Original article

A semi-discrete central scheme for the approximation of two-phase flows in three space dimensions

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Abstract

We present a new second-order (in space), semi-discrete, central scheme for the approximation of hyperbolic conservation laws in three space dimensions. The proposed scheme is applied to a model for two-phase, immiscible and incompressible displacement in heterogeneous porous media. Numerical simulations are presented to demonstrate its ability to approximate solutions of hyperbolic equations efficiently and accurately in petroleum reservoir simulations.

MSC: 35L65; 65M06

Keywords: Hyperbolic conservation laws; Central schemes; Two-phase flows

1. Introduction

We consider a model for two-phase, incompressible, immiscible displacement in heterogeneous porous media. In such a model, the highly nonlinear equations are of very practical importance [11,21,12]. In this paper we present a new semi-discrete central scheme for the approximation of the hyperbolic conservation law arising in two-phase, three-dimensional flows in heterogeneous porous media.

The conventional theoretical description of two-phase flows in a porous medium, in the limit of vanishing capillary pressure, is via Darcy's law coupled to the Buckley–Leverett equation. We refer the two phases, water and oil, by the subscripts w and o, respectively. We also assume that the two fluid phases saturate the pores. With no sources or sinks, and neglecting the effects of gravity, these equations become

$$\nabla \cdot \vec{v}_d = 0, \quad \text{where } \vec{v}_d = -\lambda(s) K(\vec{x}) \nabla p, \tag{1.1}$$

and

$$\frac{\partial s}{\partial t} + \nabla \cdot (f(s)\vec{v}) = 0. \tag{1.2}$$

Here, $\vec{v}_d$ is the Darcy velocity, and the seepage velocity $\vec{v} = \vec{v}_d/\phi$, where ϕ is the porosity which is assumed to be a constant. Furthermore, s is the water saturation, $K(\vec{x})$ is the absolute permeability, and p is the pres-

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