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Parabolic equations with double variable nonlinearities

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Abstract

The paper is devoted to the study of the homogeneous Dirichlet problem for the doubly nonlinear parabolic equation with nonstandard growth conditions:

$$u_t = div\left(a(x,t,u)|u|^{\alpha(x,t)}|\nabla u|^{p(x,t)-2}\nabla u\right) + f(x,t)$$

with given variable exponents $\alpha(x,t)$ and $p(x,t)$. We establish conditions on the data which guarantee the existence of bounded weak solutions in suitable Sobolev–Orlicz spaces.

MSC: 35K57; 35K65; 35B40

Keywords: Parabolic equation; Double nonlinearity; Variable nonlinearity; Nonstandard growth conditions

1. Introduction

We study the Dirichlet problem for the doubly nonlinear parabolic equation

$$\begin{cases} u_t = div\left(a(z,u)|u|^{\alpha(z)}|\nabla u|^{p(z)-2}\nabla u\right) + f(z) \quad z=(x,t)\in Q=\Omega\times(0,T], \\ u(x,0)=u_0(x) \quad in \quad \Omega, \quad u=0 \quad on \quad \Gamma=\partial\Omega\times[0,T]. \end{cases} \tag{1.1}$$

Eq. (1.1) is formally parabolic, but it may degenerate or become singular at the points where $u=0$ or $|\nabla u|=0$. Introducing the functions

$$\gamma(z)=\frac{\alpha(z)}{p(z)-1}, \quad v(z)=\int_0^u |s|^{\gamma(z)}ds=\frac{u|u|^{\gamma(z)}}{\gamma(z)+1}, \quad u(z)=\Phi_0(z,v)=\frac{|v|^{\frac{-\gamma}{1+\gamma}}v}{(1+\gamma)^{\frac{1}{1+\gamma}}}, \tag{1.2}$$

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