Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation 81 (2010) 536–550

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

A smooth estimator for MC/QMC methods in finance

Chuan-Hsiang Han [a,*], Yongzeng Lai [b]

[a] Department of Quantitative Finance, National Tsing-Hua University, 101, Section 2, Kuang Fu Road, Hsinchu 30013, Taiwan, ROC
[b] Department of Mathematics, Wilfrid Laurier University, Waterloo, Ontario N2L 3C5, Canada

Received 24 July 2007; received in revised form 12 October 2009; accepted 16 July 2010
Available online 27 July 2010

Abstract

We investigate the effect of martingale control as a smoother for MC/QMC methods. Numerical results of estimating low-biased solutions for American put option prices under the Black–Scholes model demonstrate that using QMC methods can be problematic. But it can be fixed by adding a (local) martingale control variate into the least-squares estimator to gain accuracy and efficiency. In examples of estimating European option prices under multi-factor stochastic volatility models, randomized QMC methods improve the variance by merely a single digit. After adding a martingale control, the variance reduction ratio raise up to 700 times for randomized QMC and about 50 times for MC simulations. When the delta estimation problem is considered, the efficiency of the martingale control variate method decreases. We propose an importance sampling method which performs better particularly in the presence of rare events.

Keywords: Option pricing; Multi-factor stochastic volatility models; Control variate method; Monte Carlo and Quasi-Monte Carlo methods

1. Introduction

The evaluation of financial derivatives is a central problem in modern finance. In the seminal work of Black and Scholes [4], the fair price of a European-style derivative, denoted by P, can be presented as a conditional expectation under the risk-neutral probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\le t\le T}, \mathbb{P}^\star)$

$$P(t, S_t) = \mathbb{E}^\star\{e^{-r(T-t)}H(S_T)|\mathcal{F}_t\}, \tag{1}$$

where the underlying risky-asset S_t is governed by the geometric Brownian motion

$$dS_t = rS_t\,dt + \sigma S_t\,dW_t^\star. \tag{2}$$

Other notations are defined as follows: t the current time, $T<+\infty$ the maturity, r the risk-free interest rate, σ the volatility, $W_t^\star$ the standard Brownian motion, and $H(x)$ the payoff function satisfying the usual integrability condition. For example, if $H(x)=\max\{x-K, 0\}\equiv(x-K)^+$ for the strike price $K>0$, it is a call payoff; if $H(x)=\max\{K-x, 0\}\equiv(K-x)^+$, it is a put payoff. A financial contract with the call or put payoff exercised at the maturity date is called a European call option or a European put option respectively.

* Corresponding author. Tel.: +886 3 5742224; fax: +886 3 5715403.
E-mail addresses: chhan@mx.nthu.edu.tw (C.-H. Han), ylai@wlu.ca (Y. Lai).

0378-4754/$36.00
doi:10.1016/j.matcom.2010.07.013