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Second-order-accurate discrete ordinates solutions of transient radiative transfer in a scattering slab with variable refractive index☆

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ABSTRACT

The discrete ordinates method (DOM) with a second-order upwind interpolation scheme is applied to solve transient radiative transfer in a graded index slab suddenly exposed to a diffuse strong irradiation at one of its boundaries. The planar medium is absorbing and anisotropically scattering. From the comparison of the results obtained by the first-order DOM, the second-order DOM, the modified DOM and the Monte Carlo method, it can be seen that the numerical diffusion in the transient solutions obtained by the second-order DOM is less than that in the solutions obtained by the first-order DOM, but the numerical diffusion is still noticeable, especially for optically thin and moderate cases. By contrast, for optically thick cases the numerical diffusion due to the finite difference of the advection term of the transient radiative transfer equation is minor. In general, it is still necessary to adopt a DOM with a higher order scheme to capture the wave front of transient radiative transfer accurately. Besides, the influence of numerical diffusion is a little less noticeable for the case with a larger gradient of refractive index, and the distribution of direction-integrated intensity around the irradiation boundary decreases and that around the other boundary increases with the increase of the anisotropically scattering coefficient.

1. Introduction

Over the past few decades a considerable number of studies have been made on transient radiative transfer in scattering media [1,2]. Recent studies on transient radiative transfer were reviewed by Kim and co-workers [2]. Only a few of them considered the radiative transfer problems incorporating the spatially continuous variation of refractive index in the medium. In these situations, because of the continuous variation of the refractive index, the speed of light is a function of the position and the radiation streams in curved paths rather than straight lines. The effects of the spatially continuous variation of refractive index on radiative transfer in a scattering medium are important in numerous applications, such as the propagation of laser pulse through a cloud of aerosols [3] and the thermal barrier coatings [4]. Several variations of the transient radiative transfer equation for a scattering medium with a spatially varying refractive index have been proposed by many authors [5–7]. Among those authors, Premaratne and co-workers [7] have shown that their result reduces to the standard radiative transfer equation when the refractive index is constant and to the geometrical optics equation when the medium is lossless and non-scattering. Wu [8] and Liu and Hsu [9] used the discrete ordinates method (DOM) and the discontinuous finite element method to solve transient radiative transfer in gradient index media, respectively. Various differential approximation methods for transient radiative transfer in refractive planar media were examined [10]. Recently, a Monte Carlo method (MCM) based on the path length and time of flight in closed form was presented [11] and a modified discrete ordinates method (MDOM) was developed [12] for transient radiative transfer in a refractive slab suddenly exposed to a diffuse irradiation.

In this work, we adapt a second-order upwind scheme developed for solving Eulerian advection problems [13] to reduce numerical diffusion in the discrete ordinates solutions of transient radiative transfer in a refractive slab. The planar medium considered is anisotropically scattering and is suddenly exposed to a diffuse irradiation at one of its boundaries. Because the exact analytical path length in a slab with a linear refractive index is available [14], accurate solutions of transient radiative transfer in such a medium can be obtained by the MCM. Thus, for comparison purpose, we consider the case with a linear refractive index in this work. By comparing the present solutions with the results obtained by the MCM, the first-order DOM and the MDOM, we examine the performance of the second-order-accurate DOM. Because of the steep features of the transient

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