International Communications in Heat and Mass Transfer 38 (2011) 418–424

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

A sensitivity analysis for the determination of unknown thermal coefficients through a phase-change process with temperature-dependent thermal conductivity ☆

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ARTICLE INFO

Available online 8 January 2011

Keywords:
Stefan problem
Unknown thermal coefficients
PCM
Sensitivity analysis
Inverse problem
Free boundary problem
Moving boundary problem

ABSTRACT

In Tarzia, Int. Comm. in Heat and Mass Transfer, 25 (1998), 139–147, explicit formulas for the simultaneous determination of unknown thermal coefficients of a semi-infinite material through a phase-change process with temperature-dependent thermal conductivity were obtained. Moreover, ten different cases were studied: four cases of free boundary problems (i.e. Stefan-like problems) and six cases of moving boundary problems (i.e. inverse Stefan-like problems).

The goal of this paper is to obtain a numerical sensitivity analysis of the mentioned ten cases for the simultaneous determination of unknown thermal coefficients and to determine the coefficients which are more sensitive with respect to the given parameters. We show numerical result for the aluminum.

1. Introduction

Heat transfer problems with a phase-change such as melting and freezing have been studied in the last century due to their wide scientific and technological applications. A review of a long bibliography on moving and free boundary problems for phase-change materials (PCM) for the heat equation is shown in [16].

We consider the following solidification problem for a semi-infinite material with an over specified condition on the fixed face $x=0$ [1,3,4,7]:

$$\begin{cases} i)\ \rho c T_t(x,t) = (k(T)T_x(x,t))_x, \quad 0<x<s(t),\ t>0 \\ ii)\ T(0,t) = T_o < T_f \ , \quad t>0 \\ iii)\ k(T_o)T_x(0,t) = \dfrac{q_o}{\sqrt{t}} \ , \quad t>0 \quad , \quad q_o>0 \\ iv)\ T(s(t),t) = T_f \ , \quad t>0 \\ v)\ k\left(T_f\right)T_x(s(t),t) = \rho h \dot{s}(t), \quad t>0 \end{cases} \tag{1}$$

where $T(x,t)$ is the temperature of the solid phase, $\rho>0$ is the density of mass, $h>0$ is the latent heat of fusion by unity of mass, $c>0$ is the specific heat, $x=s(t)$ is the phase-change interface, T_f is the phase-change temperature, T_o is the temperature at the fixed face $x=0$ and q_o is the coefficient that characterizes the heat flux at $x=0$ given by Eq. (1*iii*), which must be obtained experimentally through a phase-change process [2]. We suppose that the thermal conductivity has the following expression [5]:

$$k = k(T) = k_o\left[1 + \beta(T - T_o)/\left(T_f - T_o\right)\right], \quad \beta\in\mathbb{R}. \tag{2}$$

Let $\alpha_o = k_o/\rho c$ be the coefficient of the diffusivity at the temperature T_o. We observe that if $\beta=0$, the problem (1) becomes the classical one-phase Lamé-Clapeyron-Stefan problem with an overspecified condition at the fixed face $x=0$, and for this problem the corresponding simultaneous determination of thermal coefficients was studied in [13,14]. The phase-change process with temperature-dependent thermal coefficient of the type (2) was firstly studied in [5]. Other papers related to determination of thermal coefficients are [8,10,11,17–20].

The solution to problem (1) is given by [5,15]:

$$\begin{cases} i)\ T(x,t) = T_o + \dfrac{\left(T_f - T_o\right)}{\Phi(\lambda)}\Phi(\eta), \quad \eta = \dfrac{x}{2\sqrt{\alpha_o t}} \quad , \quad 0<\eta<\lambda \\ ii)\ s(t) = 2\lambda\sqrt{\alpha_o t} \end{cases} \tag{3}$$

where $\Phi = \Phi(x) = \Phi_\delta(x)$ is the modified error function, for a given $\delta > -1$, the unique solution to the following boundary value problem in variable x, i.e:

$$\begin{cases} i)\ [(1+\delta\ \Phi'(x))\Phi'(x)]' + 2x\Phi'(x) = 0 \ , \quad x>0, \\ ii)\ \Phi(0^+) = 0 \quad , \quad \Phi(+\infty) = 1 \end{cases} \tag{4}$$

☆ Communicated by W.J. Minkowycz.

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0735-1933/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2010.12.017