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Axially symmetric problems of heat conduction in a periodically laminated layer with vertical cylindrical hole ☆

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ABSTRACT

The distributions of temperature and heat fluxes in a periodically laminated layer with a vertically located cylindrical hole are obtained approximately within the homogenized model with microlocal parameters. The method of solution is based on the Weber–Orr integral transforms. The effects of geometrical and mechanical parameters of the composite structure on the thermal fields are discussed and presented in graphical form.

1. Introduction

Periodically layered composite materials can be made by man (laminated composites) or can be found in nature (varved clays, sandstone-slates, sandstone-shales, thin-layered limenstones). Determination of a temperature, heat flux distributions in laminated composites is very important in some problems of civil engineering, mechanical engineering. The knowledge of thermal fields in the nonhomogeneous bodies with holes is used in mining engineering, engineering geology, and geophysics.

This paper deals with the stationary heat conduction problems for a periodically laminated layer with vertically located cylindrical hole. The classical descriptions of the problems are connected with partial differential equations on discontinuous or rapidly oscillating coefficients. In the case of large number of repeated layers being components of composite the compliance of continuity conditions on interfaces leads to complicated problems for direct analytical and numerical approaches. For these reasons, structures of this kind are usually analyzed by using some approximated models. One of them is the homogenized model with microlocal parameters [1,2]. The governing equations of the homogenized model are expressed in terms of unknown macrotemperature and certain extra unknown called the microlocal parameter. It should be emphasized that the continuity conditions on interfaces are fulfilled within the homogenized model. This model has been applied to obtain solutions to many problems (see, for a partial review, ref. [3]). The results obtained within the framework of the homogenized model were compared with the solutions of the classical heat condition in [4–6], where good consistencies of temperature and heat fluxes within the both approaches for some problems of semi-infinite layer have been stated.

In the presented paper axially symmetric stationary problems of heat conduction in a periodically laminated layer with vertically located cylindrical hole is considered. The body is assumed to be composed of periodically repeated two-layered fundamental lamina. The perfect thermal contact between the layers is taken into account. The cylindrical hole is located perpendicular to the boundary planes and two cases of boundary conditions on the lateral surface of the hole are considered: (1^0) zero temperature (Problem (a)), (2^0) zero radial component of heat flux (Problem (b)). In the both cases the lower surface is kept at zero temperature and the upper surface is kept at a known temperature dependent on the radius. The above formulated problems are solved within the framework of the homogenized model with microlocal parameter [1–6] and by using the Weber–Orr integral transforms suitable for the considered boundary value problems. These transforms were introduced firstly by Weber and Orr; however, the exact proofs of the forms of inverse transforms and the transform properties were presented in refs. [7–9]. Their applications to problems of mechanics can be found in refs. [10–12].

2. Formulations and solutions of the problems

Consider a rigid, periodically stratified layer of the thickness h with a cylindrical hole normal to the layering (Fig. 1). The radius of the hole is denoted by a and the thicknesses of the subsequent layers are marked by l_1, l_2, so $l = l_1 + l_2$ is the thickness of the fundamental layer. The constituents of the considered body are isotropic and homogeneous heat conductors. Let K_1, K_2 denote the coefficients of heat conductivities of the subsequent layers: Let (r, φ, z) be the cylindrical coordinate system such that $z = 0$ and $z = h$ are the boundary surfaces of the body. Moreover, the axis z represents the symmetry axis of the hole. The boundary surface is assumed to be kept at given temperature

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