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The method of fundamental solutions for solving free boundary saturated seepage problem☆

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ABSTRACT

The problem of seepage flow through a dam is free boundary problem that is more conveniently solved by a meshless method than a mesh-based method such as finite element method (FEM) and finite difference method (FDM). This paper presents method of fundamental solutions, which is one kind of meshless methods, to solve a dam problem using the fundamental solution to the Laplace's equation. Solutions on free boundary are determined by iteration and cubic spline interpolation. The numerical solutions then are compared with the boundary element method (BEM), FDM and FEM to display the performance of present method.

1. Introduction

The two-dimensional steady state saturated isotropic seepage flow with free boundary is described by the Laplace equation necessary boundary conditions. Previous works, the methods are to solve the unconfined seepage problem; it can be classified as analytical and numerical methods. The analytical solution can be obtained by using the theory of analytical function for liner ordinary differential equations [1,2]. It is only valid for two-dimensional problem but it cannot be used in case of complex geometrics and three-dimensional problems.

Conventionally numerical methods used to solve such problem included FDM [3] and FEM [4–8]. These methods are all mesh-dependent methods because they require boundary-fitted mesh generation. Alternative numerical methods include BEM [9] and MFS [10,11]. Both methods do not require boundary-fitted mesh, which results in considerable simplification of the preprocessing step. MFS has additional advantages over BEM in that it requires only boundary node placement instead of boundary mesh generation, and it does not require evaluation of near singular integrals [12]. The basic idea of MFS is to approximate the solution by forming a linear combination of known fundamental solutions with sources located outside the problem domain.

Previously, Chantasiriwan [13] investigated numerically both one- and two-phase Stefan problem subject to specification of boundary temperature, heat flux or energy using MFS. The numerically obtained results showed good agreement with the available analytical solutions. Kolodziej et al. [11] implemented the MFS with radial basis functions to solve a heat source problem for arbitrary domains, the numerical results showed that the MFS is an accurate and reliable numerical technique for the solution of the inverse heat source problem.

In order to study seepage problem, accurately defining the position of free boundary is very important and necessary. In the past, many researchers utilized several methods to determine the location of free boundary such as Aitchison [3], and Westbrook [4] used FDM and FEM respectively, to solve the position of the free boundary, respectively. The conventional BEM was then used to study the seepage flow through the porous media by Liggett and Liu [14], and also BEM using hypersingular equations was proposed by Chen et al. [15].

In this paper, free boundary is regarded as a moving boundary with the over-specified boundary conditions, and MFS is used to find the location of free boundary. The numerical results of present method are also compared with FDM, FEM, and BEM solutions.

2. Mathematical formulations

The seepage problem of water flow through a saturated porous medium dam with tail water is shown in Fig. 1. The free boundary is defined as the boundary line or interface between the saturated-wet and dry soils. In order to reduce complexity of the phenomena to analyze flow field in the soil, several assumptions are introduced as following:

(1) Soil in the flow field is homogeneous and isotropic.
(2) Capillary and evaporation effects are neglected.
(3) Two dimensional steady-state flow.
(4) The flow follows Darcy's law.
(5) Hydraulic conductivity or permeability of the soil is constant isotropic seepage flow.

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