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Steady flow and heat transfer of the power-law fluid over a rotating disk ☆

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ABSTRACT

This paper deals with the steady flow and heat transfer of a viscous incompressible power-law fluid over a rotating infinite disk. Assumed the thermal conductivity follows the same function as the viscosity, the governing equations in the boundary layer are transformed into a set of ordinary differential equations by generalized Karman similarity transformation. The corresponding nonlinear two-point boundary value problem was solved by multi-shooting method. Numerical results indicated that the parameters of power-law index and Prandtl number have significant effects on velocity and temperature fields. The thickness of the boundary layer decays with power-law index. The peak of the radial velocity changes slightly with power- law index. The values near the boundary are affected dramatically by the thickness of the boundary layer. With the increasing of the Prandtl number the heat conducts more strongly.

1. Introduction

The incompressible fluid flow and heat transfer over rotating bodies have many applications and have been studied in many industrial, geothermal, geophysical, technological and engineering. For the complex of real machine, this type of flow can be modeled by the rotating disk system. The steady flow of Newtonian fluid over rotating disk was first discussed by von Karman in 1921 [1]. Von Karman introduced an elegant transformation reduced the Navier–Stokes equations to a set of ordinary differential equations (ODEs), and obtained an approximate solution to the ODEs using momentum integral method. In 1934, Cochran [2] calculated more accurate values by numerical integration of the ODEs. In 1960 Rogers and Lance [3] and in 1966 Benton [4] obtained improved solutions. The problem of heat transfer over a rotating disk maintained at a constant temperature was first considered by Millsaps and Pohlhausen [5] with a variety of Prandtl numbers in the range of $0.5<(c_v/c_p)\mathrm{Pr}<10$ in 1952. Sparrow and Gregg [6] obtained the results for all Prandtl numbers in 1959. Considerable attentions have been devoted to the flow and heat transfer over rotating disk near Newtonian fluid. The review of Zandbergen and Dijkstra [7] provides a useful survey.

Non-Newtonian fluid is more familiar and significant. Some examples are polymer solutions and melts, rubber, grease and blood. Attia [8] and Sahoo [9] and Osalusi [10] provided some researches about the Reiner–Rivlin model. Rashaida [11] studied the flow of the Bingham fluid. A large number of fluids exhibit shear thinning and shear thickening characteristics, which are called power-law fluid. In 1964 Mitschka [12] generalized the Von Karman's similarity transformation to power-law fluid. In 2001 Andersson [13] analyzed the flow in the boundary layer of the power-law fluid systematically and extended the power-law index to $1.5\leq n\leq 2.0$.

In this paper, the steady flow and heat transfer of power-law fluid over a free rotating disk are considered. On the assumption that the thermal conductivity depends on the flow field, the coupled governing equations are transformed into ODEs in the boundary layer. Multi-shooting method is applied to solve the ODEs.

2. Physical model and mathematic equations

Let us consider the laminar flow driven by an infinite disk rotating steadily with angular velocity Ω about the z-axis. The fluid occupies the infinite region on one side of the disk. There are no-slid and impermeability on the disk. The disk maintains constant temperature T_w and the fluid out of the boundary layer keeps at a uniform temperature T_∞. The flow is steady and axial-symmetric. The cylindrical polar coordinate system is (r,φ,z), physical model is shown in Fig. 1.

From the momentum balance in the axial direction Andersson and Korte and Meland derived $\partial p/\partial z=0$ in the boundary layer. Von Karman's original similarity transformation implies $\partial p/\partial r=0$. So the pressure is considered as constant in the boundary layer. The governing equations are

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

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