



# Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field<sup>☆</sup>

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## ABSTRACT

In this paper, we examine the convective flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of a magnetic field. The governing partial differential equations with the auxiliary conditions are reduced to ordinary differential equations with the appropriate corresponding conditions via scaling transformations. The analytical solutions of the resulting ODEs are obtained, and from which the analytical solutions of the original problem are presented. The influence of pertinent parameters such as the magnetic parameter, the solid volume fraction of nanoparticles and the type of nanofluid on the flow, heat transfer, Nusselt number and skin friction coefficient is discussed. Comparison with published results is presented.

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## 1. Introduction

The study of magnetic field effects has important applications in physics, chemistry and engineering. In recent years, we find several applications in the polymer industry (where one deals with stretching of plastic sheets) and metallurgy where hydro-magnetic techniques are being used. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. Mention may be made of drawing, annealing, and thinning of copper wires. In all these cases, the properties of the final product depend to a great extent on the rate of cooling by drawing such strips in an electrically conducting fluid subject to a magnetic field and the characteristic desired in the final product. In view of these applications Pavlov [1] investigated the flow of an electrically conducting fluid caused solely by the stretching of an elastic sheet in the presence of a uniform magnetic field. Andersson [3] extended the work of Chakrabarti and Gupta [2] to the MHD flow of a non-Newtonian viscoelastic fluid over an impermeable stretching sheet and found that the magnetic parameter has the same effect as the viscoelastic parameter. Ishak et al. [4] studied the effect of a uniform transverse magnetic field on the stagnation point flow toward a vertical stretching sheet. Raptis and Perdakis [5] investigated the effect of a

chemical reaction of an electrically conducting viscous fluid on the flow over a non-linearly (quadratic) semi-infinite stretching sheet in the presence of a constant magnetic field which is normal to the sheet. Prasad et al. [6] studied the effects of variable viscosity and variable thermal conductivity on the hydro-magnetic flow and heat transfer over a non-linear stretching sheet. Hayat et al. [7] investigated the two-dimensional mixed convection boundary layer magnetohydrodynamic (MHD) stagnation-point flow through a porous medium bounded by a stretching vertical plate with thermal radiation. They considered that the stretching velocity and the surface temperature are assumed to vary linearly with the distance from the stagnation-point. Abel et al. [8] presented a mathematical analysis for the momentum and heat transfer characteristics of the boundary layer flow of an incompressible and electrically conducting viscoelastic fluid over a linear stretching sheet. The momentum boundary layer equation includes both the effect of transverse magnetic and electric fields. Kumari and Nath [9] have studied the unsteady magnetohydrodynamic flow and heat transfer of a Newtonian fluid induced by an impulsively stretched plane surface in two lateral directions by using an analytic technique, namely, the homotopy method. Further, Kumaran et al. [10] investigated the transition effect of the boundary layer flow: (1) due to a suddenly imposed magnetic field on the viscous flow past a stretching sheet and (2) due to sudden withdrawal of a magnetic field on the viscous flow past a stretching sheet under a magnetic field. In both cases the sheet stretches linearly along the direction of the fluid flow. Prasad et al. [11] analyzed the momentum and heat transfer characteristics of the boundary layers of an incompressible electrically conducting fluid flow of a viscoelastic fluid over a stretching sheet.

Nanofluid is envisioned to describe a fluid in which nanometer-sized particles are suspended in convectional heat transfer basic

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