Contents lists available at ScienceDirect



International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

# Unsteady MHD flow and heat transfer with viscous dissipation past a stretching sheet $\stackrel{\bigstar}{\rightarrowtail}$

### V. Kumaran <sup>a,\*</sup>, A.K. Banerjee <sup>a</sup>, A. Vanav Kumar <sup>a</sup>, I. Pop <sup>b</sup>

<sup>a</sup> Department of Mathematics, National Institute of Technology, Tiruchirappalli - 620015, India
<sup>b</sup> Faculty of Mathematics, University of Clui, R-3400 Clui, CP 253, Romania

#### ARTICLE INFO

Available online 2 December 2010

Keywords: Unsteady boundary layer flow Stretching sheet Magnetic field Finite difference scheme Dissipation

#### ABSTRACT

In this paper a study is carried out to analyze the unsteady heat transfer effects of viscous dissipation on the steady boundary layer flow past a stretching sheet with prescribed constant surface temperature in the presence of a transverse magnetic field. The sheet is assumed to stretch linearly along the direction of the fluid flow. The assumed initial steady flow and temperature field neglecting dissipation effects becomes transient by accounting dissipation effects when time t' > 0. The temperature and the Nusselt number are computed numerically using an implicit finite difference method. The obtained steady temperature field with dissipation is of practical importance.

© 2010 Elsevier Ltd. All rights reserved.

HEAT and MASS

#### 1. Introduction

Crane [4] gave an analytic solution for a steady flow past a linearly stretching sheet. In his paper he also addressed the heat transfer problem. Followed by this, many papers on steady stretching sheet problems are studied. It is worth mentioning that the momentum and heat transfer in an MHD Newtonian and non-Newtonian fluid flows over a stretching sheet have been studied extensively in the recent vears because of its ever increasing usage in many practical applications, see Abel et al. [16], Fang and Zhang [17], Ray Mahapatra et al. [18], Noor et al. [19], Prasad et al. [20] etc. Few papers on unsteady stretching sheet problems are also published in the literature. In most of these papers unsteadiness is due to time dependent stretching rate as studied by Chen [5], Abel et al. [7], Ishak et al. [8], Andersson et al. [9] and Tsai et al. [11], or, impulsively started stretching as in Mehmood et al. [1], Wang et al. [2], Kumari and Nath [6] and Liao [3]. Recently, Kumaran et al. [13] considered a step change in the magnetic field with time over a boundary layer flow past a stretching sheet. In the present study we consider, a novel unsteady problem, namely the initial steady heat transfer in an MHD flow past a stretching sheet, becomes transient due to the consideration of viscous dissipation when time t' > 0. Although the dissipation mechanism is in-built in the heat transfer process, in order to study the transient dissipation effect alone theoretically, the transient heat transfer due to dissipation is considered. However, the present

E-mail address: kumaran@nitt.edu (V. Kumaran).

solution when  $t' \rightarrow \infty$  (the steady boundary layer flow and heat transfer past a stretching sheet with dissipation) is of practical importance. An important effect which bears great importance on heat transfer is the viscous dissipation. When the viscosity of the fluid and/or the velocity gradient is high, the dissipation term becomes more important although it disappears at infinity. Consequently, the effects of viscous dissipation are also included in the energy equation. For many cases, such as polymer processing, which is at very high temperature and flows appeared in glacier physics, viscous dissipation plays an important role (see Cortell [14]). On the other hand, numerical results confirm that, in the laminar forced convection in straight microchannels, both the temperature dependence of viscosity and viscous dissipation effects cannot be neglected in a wide range of real operative circumstances (see Celata et al. [15]).

#### 2. Formulation

## 2.1. Initial steady state flow and heat transfer $(t' \le 0)$ – the solution of Char [10]

Consider a laminar steady boundary layer flow of an incompressible electrically conducting Newtonian fluid past a stretching sheet. The sheet issues from a thin slit at x' = 0, y' = 0, where x'-axis is along the horizontal direction of flow, y'-axis is normal to the flow, u' is the horizontal velocity component and v' is the vertical velocity component. The stretching speed is proportional to the distance from the origin and the stretching rate is  $\beta(>0)$ . The fluid is under the influence of the magnetic field of strength  $B_0$ , which acts in the direction normal to the stretching sheet. The sheet is assumed to be at temperature  $T'_{w}$  far away

<sup>🖄</sup> Communicated by A.R. Balalerishnan

<sup>\*</sup> Corresponding author.

<sup>0735-1933/\$ -</sup> see front matter © 2010 Elsevier Ltd. All rights reserved. doi:10.1016/j.icheatmasstransfer.2010.11.011