International Communications in Heat and Mass Transfer 38 (2011) 405–409

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Heat convection length for boundary-layer flows ☆

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ARTICLE INFO

Available online 8 January 2011

Keywords:
Heat convection length
Laminar
Boundary layer

ABSTRACT

In analyses of heat convection problems, it is traditional to introduce the heat transfer coefficient, h, such that the average heat flux at the solid surface can be conveniently calculated. To find h, one has to simultaneously solve governing equations for conservations of mass, momentum, and energy. This task may prove too challenging for contemporary undergraduate students. In the present study, we propose a heat convection length, Δs, which can greatly simplify the analysis, yet allow the convection characteristics to be retained. Classical examples for laminar boundary-layer air flows driven by forced convection or free convection over flat plates with length L are presented. For forced convection, $\Delta s/L$ is found to be 2.026 for a wide range of Re; for free convection, $\Delta s/L$ is found to be 2.511 for various Gr values.

1. Motivation and introduction

For problems of 2D incompressible, laminar boundary-layer air flows over a flat plate, usually one needs to solve simultaneously the continuity equation, momentum equation in the x direction, and the energy equation, which are given as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\mu\frac{\partial^2 u}{\partial y^2}+(\rho_\infty-\rho)g, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the standard boundary conditions:

$u=v=0$ and $T=T_s$ at $y=0$;
$u=u_\infty$, $v=0$, and $T=T_\infty$, at $y\to\infty$; and
$u=u_\infty$, $v=0$, and $T=T_\infty$, at $x=0$.

For forced convection, the buoyancy term in Eq. (2) can be neglected. For free convection cases with Boussinesq approximation, ρ and T are related by $\rho T=\rho_\infty T_\infty$. The positive directions of x coordinate and u velocity for this case are traditionally in the vertically upward direction.

To obtain $u(x,y)$, $v(x,y)$, and $T(x,y)$, we generally rely on a certain mathematical formulation (such as the similarity transformation) to obtain a transformed ordinary differential equation, for example, the Blasius equation for forced convection cases [1]. Such an equation is then solved using some numerical methods (such as the finite difference method and the Newton–Raphson method).

After $u(x,y)$, $v(x,y)$, and $T(x,y)$ are found numerically, the local heat transfer coefficient, h, can be determined from the relation:

$$-k\left(\frac{\partial T}{\partial y}\right)_{y=0}=h(T_s-T_\infty). \tag{4}$$

Subsequently, by varying Re and Pr, one can obtain the following correlations for the average Nusselt number

$$\overline{Nu}=0.664\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3} \tag{5a}$$

for forced convection cases [2], and

$$\overline{Nu}=\frac{4}{3}\left(\frac{Gr}{4}\right)^{0.25}G_1(\mathrm{Pr}) \tag{5b}$$

for free convection cases [3], where

$$G_1(\mathrm{Pr})=0.75\mathrm{Pr}^{0.5}\left(0.609+1.221\mathrm{Pr}^{0.5}+1.238\mathrm{Pr}\right)^{(-0.25)}.$$

While it may be manageable for experienced researchers to perform the procedure described above, it can be difficult for contemporary undergraduate students, and sometimes even graduate students, to achieve the same. Furthermore, the total time required to teach the material (from Eqs. (1)–(3) to Eqs. (5a, 5b)) may be as much as 7 or 8 lecture hours. A typical undergraduate heat transfer course

☆ Communicated by W.J. Minkowycz.

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0735-1933/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2010.12.026